SPINOFFS

Spinoffs are relatively short learning modules inspired by the LTAs. They can be easily implemented to support student learning in courses ranging from prealgebra through calculus. The Spinoffs typically give students an opportunity to use mathematics in a real world context.

LTA - SPINOFF 8A — Exploring Shuttle Preparations: Crawler and VAB

LTA - SPINOFF 8B — Unit Analysis

LTA - SPINOFF 8C — Pressure and Volume

LTA - SPINOFF 8D — At What Cost? High Versus Low Pressure K - Bottles for the Space Shuttle

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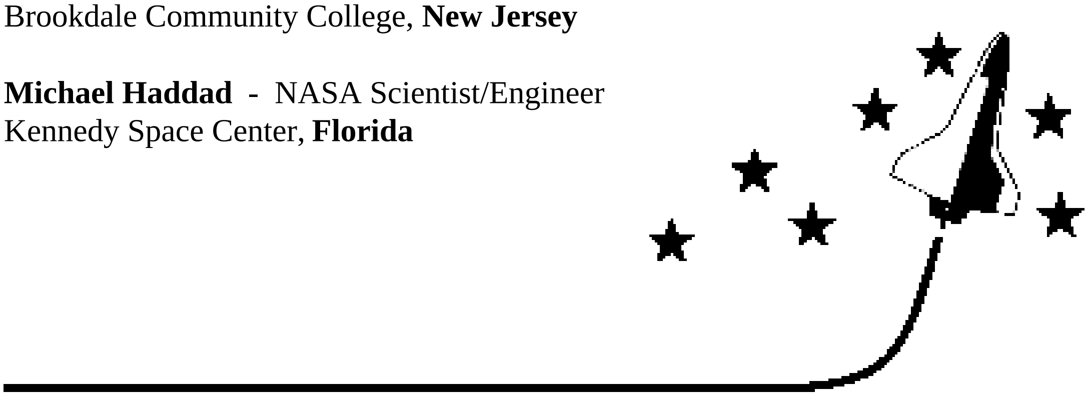

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SPINOFF 8B

Unit Analysis

Converting from one unit of measurement to another or from one rate to another is a necessary and critical part of scientific work. You may have used simple divisions or multiplications to change units before, or you may have used proportions. Another method that is often quicker and easier is called **unit analysis**. Unit analysis involves using simple equations that relate different types of measurements. For example, you know that:

12 inches = 1 foot
16 ounces = 1 pound
60 seconds = 1 minute

1) Write two other equations that you know:

These equations can be manipulated to produce equations that are equivalent to 1. Take the equation 12 inches = 1 foot, and divide both sides by one foot. This will produce:

$$\frac{12 \text{ inches}}{1 \text{ foot}} = \frac{1 \text{ foot}}{1 \text{ foot}}$$

Since the right side of the equation involves dividing a quantity by itself, we know we can rewrite the equation as:

$$\frac{12 \text{ inches}}{1 \text{ foot}} = 1$$

We often call the fraction $\frac{12 \text{ inches}}{1 \text{ foot}}$ a **unit fraction**, indicating that it is actually equivalent to 1.

You could just as easily have divided both sides of the equation, 1 foot = 12 inches, by 12 inches, and produced the equation:

$$\frac{1 \text{ foot}}{12 \text{ inches}} = 1$$

The equation 16 ounces = 1 pound can be used to derive these two unit fractions:

$$\frac{16 \text{ ounces}}{1 \text{ pound}} \text{ and } \frac{1 \text{ pound}}{16 \text{ ounces}}$$

2) Write the two unit fractions given by the equation, 60 seconds = 1 minute:

3) Write the two unit fractions given by each of the equations you gave in problem 1.

You may well be wondering why this idea is important. Remember that you can multiply any number by 1 or **an equivalent form of 1** (unit fractions *are* equivalent to 1) and the value of the number will not change. For example,

$$5 \times 1 = 5 \quad \text{and} \quad 5 \times \frac{8}{8} = \frac{40}{8} \text{ which is equivalent to 5}$$

When we multiply by unit fractions, we are multiplying by an equivalent form of 1. Because of the properties of fraction multiplication, some of the units of measure may cancel, so that the answer is in a different unit of measure. In this way, multiplying by unit fractions will help us convert from one type of measurement to another.

Let's try using these ideas to find out how many seconds there are in 5 hours. There will be two steps that we go through, first changing from hours to minutes, and then changing from minutes to seconds.

Since we first want to change from hours to minutes, we have a logical choice of two unit fractions:

$$\frac{1 \text{ hour}}{60 \text{ minutes}} \quad \text{or} \quad \frac{60 \text{ minutes}}{1 \text{ hour}}$$

We decide which of these unit fractions to use by making sure that the units we do **not** want in our final answer cancel. Since the 5 hours that we started with is actually $\frac{5 \text{ hours}}{1}$, we choose the unit fraction with **hours** in the denominator so that the **hours** will cancel and we will be left with **minutes**. Here's how it looks:

$$\frac{5 \cancel{\text{hours}}}{1} \times \frac{60 \text{ minutes}}{1 \cancel{\text{hour}}} = 300 \text{ minutes}$$

Suppose that you had chosen the unit fraction $\frac{1 \text{ hour}}{60 \text{ minutes}}$ to work with. The multiplication would look like this: $\frac{5 \text{ hours}}{1} \times \frac{1 \text{ hour}}{60 \text{ minutes}}$. Can you see that none of the units cancel?

Next, we convert the minutes to seconds. We currently have 300 minutes and we want to eliminate **minutes**, so we will multiply by the unit fraction that involves minutes and seconds and which has minutes in the denominator. Here it is:

$$\frac{300 \cancel{\text{minutes}}}{1} \times \frac{60 \text{ seconds}}{1 \cancel{\text{minutes}}} = 18{,}000 \text{ seconds}$$

It is actually much easier to do all of the multiplication at once, multiplying by unit fractions until you have reached your final unit. In one line, the conversion of 5 hours to seconds looks like this:

$$\frac{5 \cancel{\text{hours}}}{1} \times \frac{60 \cancel{\text{minutes}}}{1 \cancel{\text{hour}}} \times \frac{60 \text{ seconds}}{1 \cancel{\text{minute}}} = 18{,}000 \text{ seconds}$$

This conversion process is called **unit analysis**.

4) Use **unit analysis** to convert 7200 seconds to **hours**.

Unit analysis can also be used to convert a **rate** to a different rate. For example, suppose you want to convert 60 miles per hour to feet per second. You will need to convert the miles to feet *and* the hours to seconds. To do this you need to know the following facts:

1 mile = 5280 feet
1 hour = 60 minutes
1 minute = 60 seconds

Once again, start with the given 60 miles per hour, which is written $\frac{\text{60 miles}}{\text{1 hour}}$. Then multiply by unit fractions generated from the fact list so that the unwanted units cancel and you finish with $\frac{\text{feet}}{\text{second}}$. Note that the order in which you multiply the fractions does not matter since multiplication is commutative. You can work with the distance measure first, and then the time measure, or vice versa. Here's how one solution looks:

$$\frac{60\ \cancel{\text{miles}}}{1\ \cancel{\text{hour}}} \times \frac{5280\ \text{feet}}{1\ \cancel{\text{mile}}} \times \frac{1\ \cancel{\text{hour}}}{60\ \cancel{\text{minutes}}} \times \frac{1\ \cancel{\text{minute}}}{60\ \text{seconds}} = 88\ \text{ft / sec}$$

5) Convert 110 feet per second to miles per hour.

We can also use unit analysis for area and volume conversions. Suppose you have a surface that has an area of 1872 in^2 and you want to convert this to ft^2. You know there are 12 inches in one foot, but if you want to convert from **square** inches to **square** feet, you need to use more than a single 12. There are 12×12 or 144 square inches in 1 square foot. Using unit analysis, the conversion looks like this:

$$\frac{1872\ \cancel{\text{square inches}}}{1} \times \frac{1\ \text{square foot}}{144\ \cancel{\text{square inches}}} = 13\ \text{square feet}$$

6) Convert 100 square feet to square inches.

Now suppose you have a container that has a volume of 100 ft^3, and you want to convert this to in^3. You know that there are 12 inches in one foot but if you want to convert from **cubic** feet to **cubic** inches, you need to use more than a single 12. There are $12 \times 12 \times 12$ or 1728 cubic inches in 1 cubic foot. Using unit analysis, the conversion looks like this:

$$\frac{100\ \cancel{\text{cubic feet}}}{1} \times \frac{1728\ \text{cubic inches}}{1\ \cancel{\text{cubic foot}}} = 172{,}800\ \text{cubic inches}$$

7) Convert 50 cubic feet to cubic centimeters. To do this you will need to use:

1 cubic foot = 1728 cubic inches

1 cubic inch = $2.54 \times 2.54 \times 2.54$ or 16.387064 cubic centimeters

8) Convert 56,633 cubic centimeters to cubic feet.

9) Discussion: Is there a way you can tell ahead of time whether your answer is going to have a larger or smaller numerical value? For instance, if you are converting from feet to miles, will you get a larger or smaller value? If you convert from feet to inches? Write a short paragraph explaining to a classmate how you can tell whether the value of the answer will be larger or smaller than your starting value.