

FACULTY NOTES

The LTAs and Spinoffs are designed so that each professor can implement them in a way that is consistent with his/her teaching style and course objectives. This may range from using the materials as out-of-class projects with minimal in-class guidance to doing most of the work in class. The LTAs and Spinoffs are amenable to small group cooperative work and typically benefit from the use of some learning technology. Since the objective of the LTAs and Spinoffs is to support the specific academic goals you have set for your students, the Faculty Notes are not intended to be prescriptive. The purpose of the Faculty Notes is to provide information that assists you to take full advantage of the LTAs and Spinoffs. This includes suggestions for instruction as well as answers for the exercises.



FACULTY NOTES

LTA 8

Space-Age Cleaning

Level: LTA 8 is suitable for an intermediate algebra class, a college algebra class, a technical mathematics class, or a precalculus class.

Background Skills: Students should be able to convert units (otherwise do Spinoff 8B), construct tables and graphs, and write the equation of a line.

Calculator: While a scientific calculator is sufficient, a graphing calculator provides more flexibility. Students can store values, repeat arithmetic operations, and you may want to have them enter their data into lists so they can create a scatterplot. Later, when they create their equation, they can superimpose it on the scatterplot.

Suggested Teaching Strategies: Except for occasional television exposure, our students have very little knowledge of the space program. You may want to set the stage for this activity by having students read and discuss the introductory material at the beginning of the LTA. At this point, the internet activity in Spinoff 8A would give students a more comprehensive picture of the preparations needed for a Shuttle launch. Once the students have some appreciation of the setting, you can assign Spinoff 8B, which deals with unit conversions, for homework. Since the beginning of the main LTA deals with the relationship between pressure and volume, it would be appropriate to do Spinoff 8C, a CBL activity that shows the relationship between pressure and volume, either in class as a demonstration, or in small groups. Then proceed on to the main LTA. Group work is a suitable strategy. You should monitor the groups to be sure that each group has an acceptable level of understanding, and you should be sure that each group's numbers are correct before they get too far along. Periodically, there are assignments in the activity that ask students to summarize some of the steps or to write a memo to their supervisor with an explanation. You may want to assign these exercises individually for homework to be sure that individual students have an appropriate level of understanding. At some point during the activity it would be appropriate to discuss rounding, since the normal rules of rounding cannot be used to determine the number of bottles. In addition, it could be instructive to discuss the effect of rounding to fewer places or to the nearest whole number.

Detailed Comments

Part 1: This part has a great many calculations. It is critical that students be comfortable converting from one unit to another. Students do most of the calculations in problems 1 - 8, and then go on to complete a table, plot their data points, and obtain the equation of the line that is formed. You may need to pause after Exercise 13 to help students see the "big picture" of what they have done.

Note on use of the Ideal Gas Law: In reality the Ideal Gas Law does not apply to pressures above 200 psi. For such cases a correction factor "Z" is needed, and the adjusted equation of state for the gas becomes $PV = ZnRT$. The factor Z depends on pressure, temperature, and a table for general compressibility for medium pressures. Due to the compressibility relationships between temperature and pressures for our problem, it can be shown that the value of Z is very close to 1. This implies that for the temperature conditions and pressure range involving K-bottles, the Ideal Gas Law can be assumed.

Part 2: This part illustrates one of the differences between a typical textbook problem and a more realistic one. This point should be emphasized.

Part 3: This part introduces the idea of an unanticipated delay occurring after the portable purging system is in place. You should pause after the introductory description and give the students some time to think about and develop a strategy for making the bottles last longer. The steps are given, but this is a good place for students to do some critical thinking. The equation in this part is a rational function, so this is a nice complement to the linear function developed earlier.

Comments about the Writing Questions

Part 2 Exercise 4, Part 3 Exercise 13, and Part 4 require the students to write a paragraph or two. You will need to talk to them before they do the writing, emphasizing what their responses should cover. Some guidance is given in the questions, but in our experience, students need more help here than they do with all the calculations.

Solutions

Answers to many of the exercises depend on a sequence of calculations. If an answer has been rounded to two decimal places, we have used the rounded answer in subsequent calculations. If you retain all digits provided by the calculator at each step, your answers may differ somewhat from the following.

Part 1 - A Space-Age Cleaning Problem

$$1) P_1 V_1 = P_2 V_2 \quad V_2 = \frac{P_1 V_1}{P_2} = \frac{(2200 \frac{\text{lbs}}{\text{in}^2})(1.5 \text{ ft}^3)}{14.8 \frac{\text{lbs}}{\text{in}^2}} = 222.972973 \text{ ft}^3$$

$$2) V_2 = 222.972973 \text{ ft}^3 \times \frac{1728 \text{ in}^3}{1 \text{ ft}^3} \times \frac{2.54^3 \text{ cm}^3}{1 \text{ in}^3} = 6,313,891.47 \text{ cm}^3$$

$$3) \text{ Time to use one K-bottle} = \frac{6,313,891.47 \text{ cm}^3}{100 \frac{\text{cm}^3}{\text{sec}}} = 63,138.91 \text{ sec}$$

$$4) \text{ Time- one K-bottle} = 63,138.91 \text{ sec} \times \frac{1 \text{ min}}{60 \text{ sec}} \times \frac{1 \text{ hour}}{60 \text{ min}} \times \frac{1 \text{ day}}{24 \text{ hours}} = 0.73 \text{ days}$$

$$5) \text{ Number of K-bottles} = 7 \text{ days} \times \frac{1 \text{ bottle}}{0.73 \text{ days}} = 9.59 \text{ K-bottles}$$

Whole number of bottles needed to purge for 7 days = 10 K-bottles

- 6) a) $V \div \text{flow rate} = \text{seconds}$
- b) $\text{seconds} \div (60^2 \times 24) = \text{days per bottle}$
- c) $7 \text{ days} \div \text{days per bottle} = \text{bottles}$

7. a) $6,313,891.47 \text{ cm}^3 \div 60 = 105,231.52 \text{ sec}$

b) $105,231.52 \text{ sec} \div (60^2 \times 24) = 1.22 \text{ days}$

c) $7 \div 1.22 = 5.74 \text{ bottles}$

Number of K-bottles needed for 7 days at a rate of $60 \text{ cm}^3/\text{sec} = 5.74 \text{ K-bottles}$

Whole number of K-bottles to purge for 7 days at a rate of $60 \text{ cm}^3/\text{sec} = 6 \text{ K-bottles}$

8. a) $6,313,891.47 \text{ cm}^3 \div 120 = 52,615.76 \text{ sec}$

b) $52,615.76 \text{ sec} \div (60^2 \times 24) = 0.61 \text{ days}$

c) $7 \div 0.61 = 11.48 \text{ bottles}$

Number of K-bottles needed for 7 days at a rate of $120 \text{ cm}^3/\text{sec} = 11.48 \text{ bottles}$

Whole number of K-bottles to purge for 7 days at a rate of $120 \text{ cm}^3/\text{sec} = 12 \text{ bottles}$

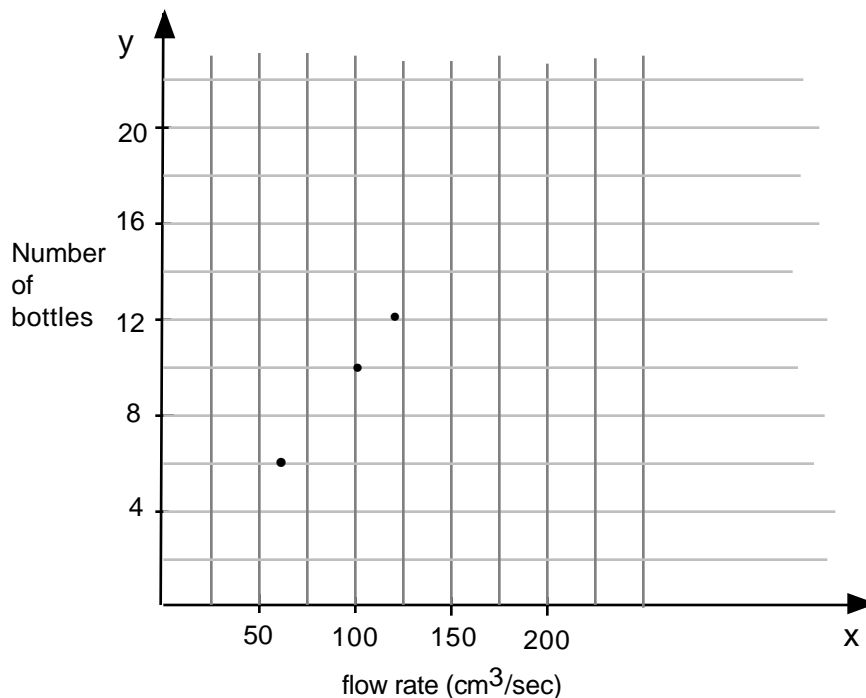
9) Summary of the results of 5, 7, and 8:

Table 1

Flow rate (cubic centimeters/second)	Whole number of K-bottles needed
60	6
100	10
120	12

10) The number of bottles is one-tenth of the flow rate. Also, for every increase of 20 in the input (flow rate), the output increases by 2 bottles.

11)

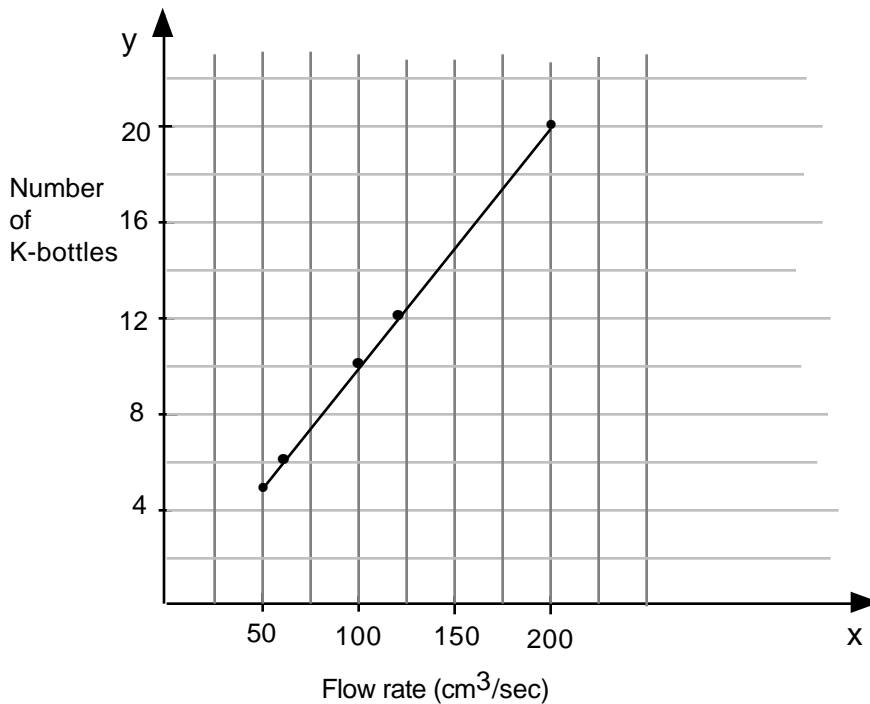


12) The relationship is linear. When the input changes by equal amounts the output changes by equal amounts. That is, $\frac{y}{x}$ is constant; $\frac{y}{x} = 0.1$.

13) $y = 0.1x$, x = flow rate, and y = number of bottles to purge for 7 days. The number of bottles needed for 7 days using a flow rate of x (cm^3/sec) is one-tenth of the flow rate. Given any flow rate, the equation can be used to find the number of bottles required to purge for 7 days.

14) $y = 0.1(50) = 5$ bottles
 $y = 0.1(200) = 20$ bottles

15)



Part 2 - A Reality Check

1) Revised formula: $y = (1.1)(0.1x) = 0.11x$

2) **Table 2**

Flow rate (cubic centimeters/second)	Number of K-bottles given by revised formula	Whole number of K-bottles needed
50	5.5	6
60	6.6	7
100	11	11
120	13.2	14
200	22	22

3) This exercise involves writing a Project Report.

Part 3 - Delays

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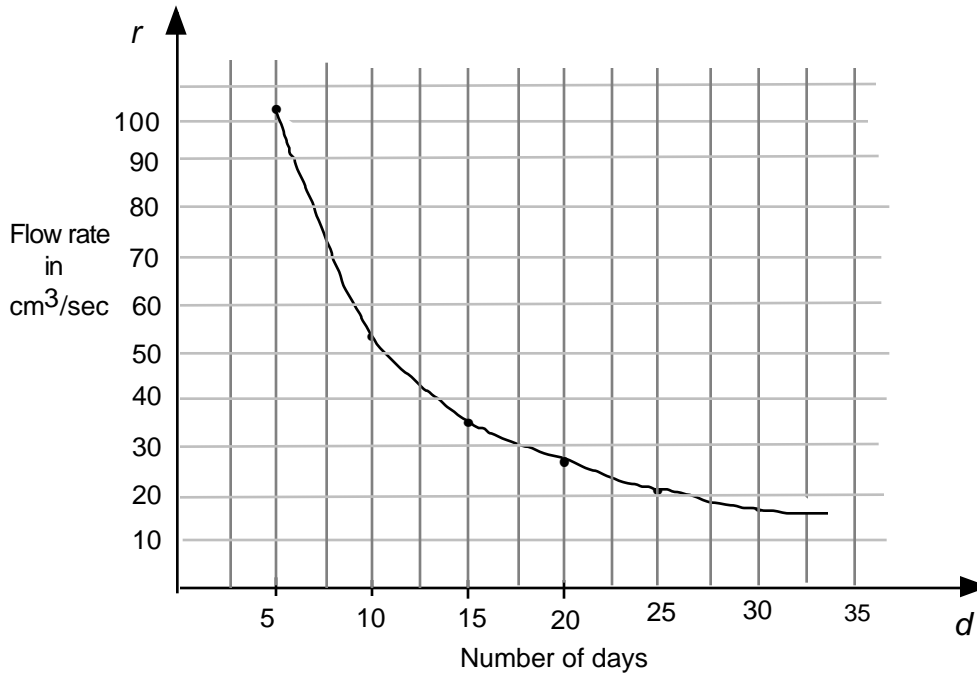
- 1) Volume of air at 14.8 lbs/in² from 11 K-bottles = 69,452,806.17 cm³
- 2) Volume of available air at 14.8 lbs/in² from 11 K-bottles = 62,507,525.55 cm³
- 3) Amount of air used per day = $\frac{62,507,525.55 \text{ cm}^3}{7 \text{ days}} = 8,929,646.51 \frac{\text{cm}^3}{\text{day}}$
- 4) Amount of air left after two days = (8,929,646.51)(5) = 44,648,232.55 cm³
- 5) $d = 44,648,232.55 \text{ cm}^3 \times \frac{1 \text{ sec}}{r \text{ cm}^3} \times \frac{1 \text{ min}}{60 \text{ sec}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ day}}{24 \text{ hr}} = \frac{516.76}{r}$
- 6) $r = \frac{516.76}{d}$

7) **Table 3**

Number of days, d	Flow rate, r (cubic centimeters/second)
5	103.35
10	51.68
15	34.45
20	25.84
25	20.67
30	17.23

- 8) As the number of days increases, the flow rate decreases, rapidly at first and then more slowly.
- 9) As you move from left to right, the graph should come down sharply and then taper off. This means that the graph should go down but bend upward.

10)



11) Table 3 indicates that the maximum purge time is slightly more than 25 days. From the graph in Exercise 10, it looks as though the maximum purge time is close to 26 days. If we let $r = 20$ in the equation $d = \frac{516.76}{r}$, then $d = 25.84$ days.

12) This memo to the supervisor will vary from student to student.

Part 4 - Wrap-Up

The answers for these questions will vary from student to student. However, all answers should represent careful reflection.