LTA 7

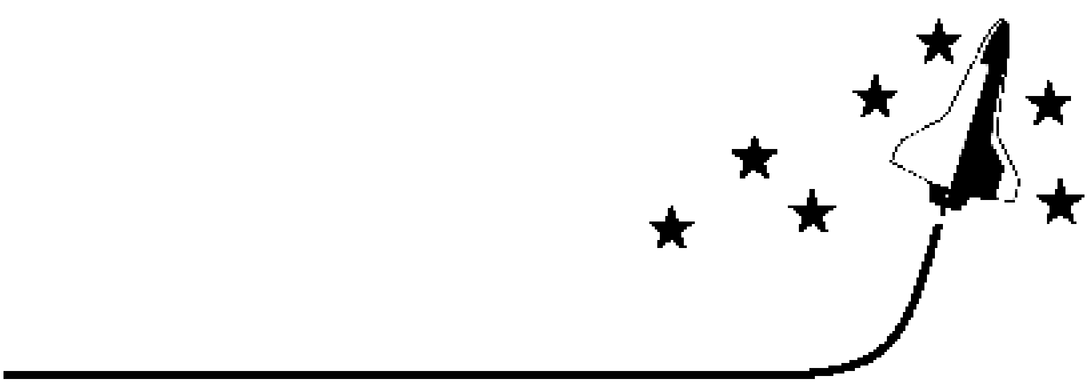
NASA - AMATYC - NSF
Project Coalition

Kennedy Space Center

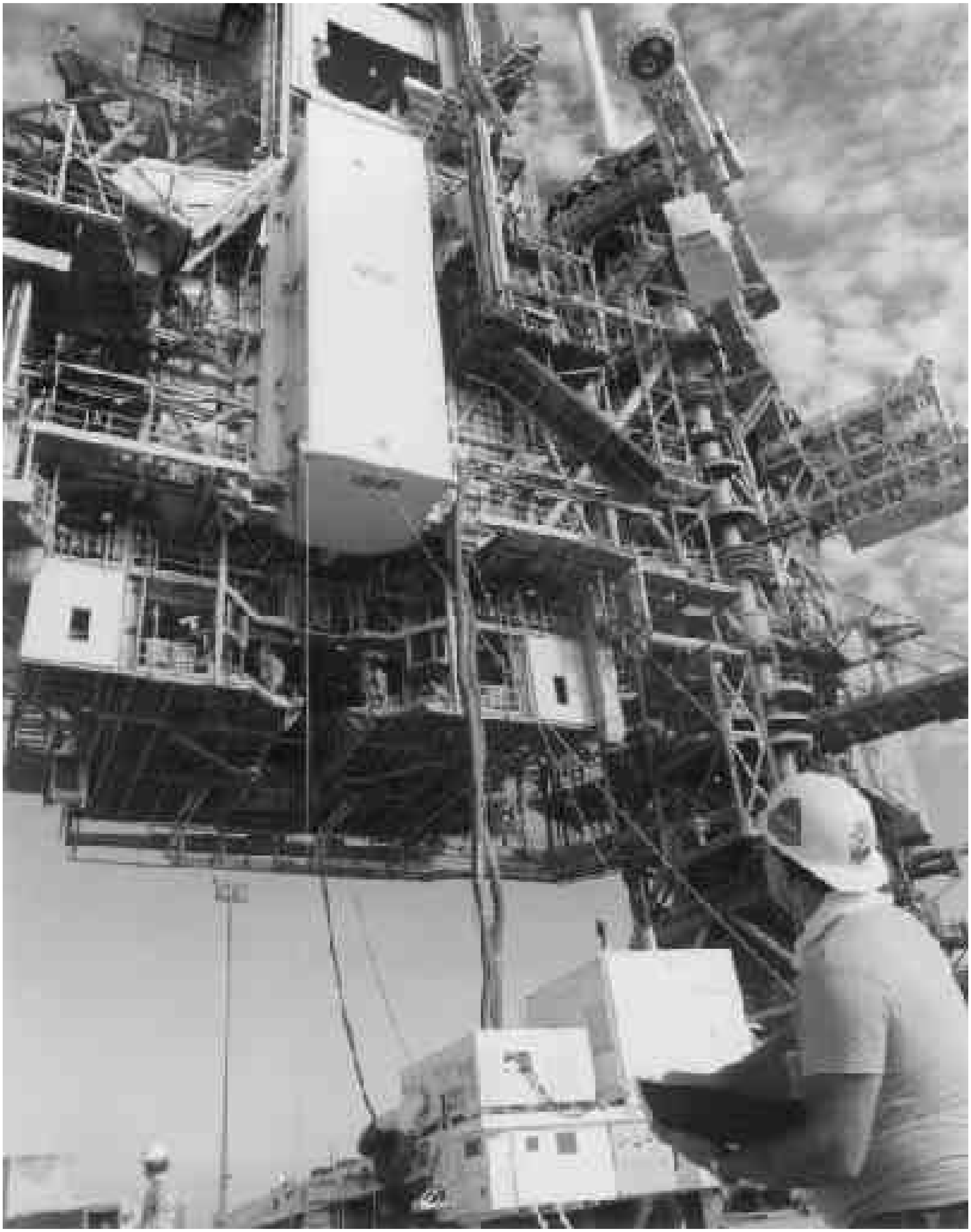
Houston We Have a Problem! -
Curve Fitting for the Spacelab

Mathematics for Engineering Technology

Electrical
Systems



Capital Community-Technical College



While in the Transport Container, the Galileo Spacecraft is installed into the Payload Changeout Room at Pad 39-B

LTA 7

Houston We Have a Problem! - Curve Fitting for the Spacelab

Mathematics for
Electrical Engineering Technology
Systems EngineeringTechnology

Julie Hess - AMATYC Writing Team Member
Montcalm Community College, Sidney, **Michigan**

Julie Hess taught at Montcalm Community College from 1994 to 1997 and is currently a faculty member at Grand Rapids Community College, Michigan. She has degrees in Operations Research and Math/Physics Education. Julie was a Founding Officer of Ohio State University's Council of Teachers of Mathematics and was selected to attend the Ohio State University Technology Strategies Workshop sponsored by the National Science Foundation.

Paul Hess - AMATYC Writing Team Member
Grand Rapids Community College, Grand Rapids, **Michigan**

Since 1993, Paul Hess has been a faculty member at Grand Rapids Community College. He served as a panelist for curriculum reform at a MichMATYC Conference and was elected a NISOD Medallion winner by fellow faculty members. Paul has been involved with numerous math solving problem competitions, has done extensive work with the TI-92™ calculator, and has received a grant to purchase calculators for his college.

Carlos Rodriguez - NASA Scientist/Engineer
Kennedy Space Center, **Florida**

In 1988, Carlos Rodriguez began work for NASA at the Space Shuttle Guidance and Navigational System Branch. From 1992 to 1997 Carlos worked as a Computer Engineer at Payloads Division where he performed database coefficients calculations and computer programming for the Spacelab Checkout System. Since 1997, Carlos has worked at the Checkout Launch and Control Systems Software Applications Division. He continues to learn about the fascinating mysteries of Mathematical Models and Linear Transformations.

Project Grant Team

John S. Pazdar
Project Director
Capital Comm-Tech College
Hartford, Connecticut

Patricia L. Hirschy
Principal Investigator
Asnuntuck Comm-Tech College
Enfield, Connecticut

Peter A. Wursthorn
Principal Investigator
Capital Comm-Tech College
Hartford, Connecticut

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LTA 7

Houston, We Have a Problem! - Curve Fitting for the Spacelab

The following project is based on actual events encountered by Carlos Rodriguez, an engineer for NASA at Kennedy Space Center in Florida.

CONGRATULATIONS!!! You have just been hired by NASA/Kennedy Space Center. Your job includes working on a team that performs testing of Spacelab experiments. Spacelab is the container that is loaded into the Space Shuttle bay which will hold the astronauts and the many experiments they will perform in space.

Because of the extreme importance of success, all experiments are tested several times on the ground before being sent into space. Your team performs the first round of testing the experiments. It involves using a computer to simulate many of the processes that will be used on the Space Shuttle when performing the experiments in space. After all, hauling the Space Shuttle into your office would be rather inconvenient.

NASA engineers have discovered that temperature readings on a power supply are not being displayed accurately by the computer. Your team's job is to investigate the current system, determine the cause for the inaccuracy, and suggest possible improvements. You know that this is extremely important, as failure could result in the loss of many important things, such as expensive equipment, valuable time, and your job.

In order for the temperature to be transmitted to and displayed by the computer, the following process must be performed (see Figure 1 below). First, a data collection device called a sensor/signal conditioner senses the temperature of the power supply and emits a corresponding voltage. This voltage is sent to an "analog to digital converter" (A/D converter) which outputs an integer measured in units called counts. Finally, the computer takes the integer and, using an equation that you provide, converts it back to the original temperature or at least to a close approximation. As a point of interest, many systems on the Space Shuttle have sensors that operate in the same fashion.

The following is an example. When the power supply has a temperature of 41.9˚C, the sensor/signal conditioner emits a voltage of 2.5 volts. The A/D converter takes that 2.5 volts and returns 512 counts. The computer then uses the equation you provide to convert 512 counts back to approximately 41.9˚C. To get this equation for the computer you can do the following: (1) Find an equation to model the conversion from temperature to voltage performed by the sensor/signal conditioner, (2) find an equation to model the conversion from voltage to counts performed by the A/D converter, and (3) use these two equations to find one single equation that the computer will use to convert counts back to temperature. The following activities will lead you through this process.

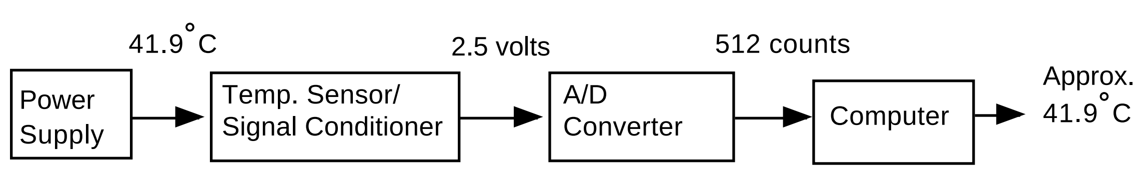


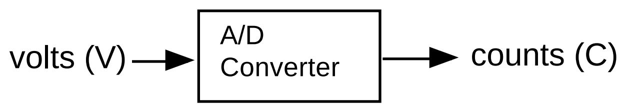
Figure 1

Section A - Volts to Counts Equation

Your team decides to first concentrate on the A/D converter because it is the easiest to deal with.

1) You have been told that this A/D converter has been configured to have an input minimum of 0 volts which gives an output of 0 counts. It has an input maximum of 5 volts which gives an output of 1024 counts. You know that the A/D conversion is linear. Determine which variable, voltage or counts, should be on the x-axis and which should be on the y-axis. Sketch a graph of the A/D conversion.

2) The manufacturer of the A/D converter does not provide the equation of this line because the converter can be configured in many different ways. Find the equation of the line using the two "points" given in Exercise 1 above. (Please use variables that identify the quantities, such as C for Counts and V for volts.)



3) Figure 1 shows that the A/D converter sends an input of 2.5V to an output of 512 counts. Explain why this input-output pair helps to confirm that the conversion from volts to counts is linear.

4) Complete the following table using your results from either Exercise 1 or 2.

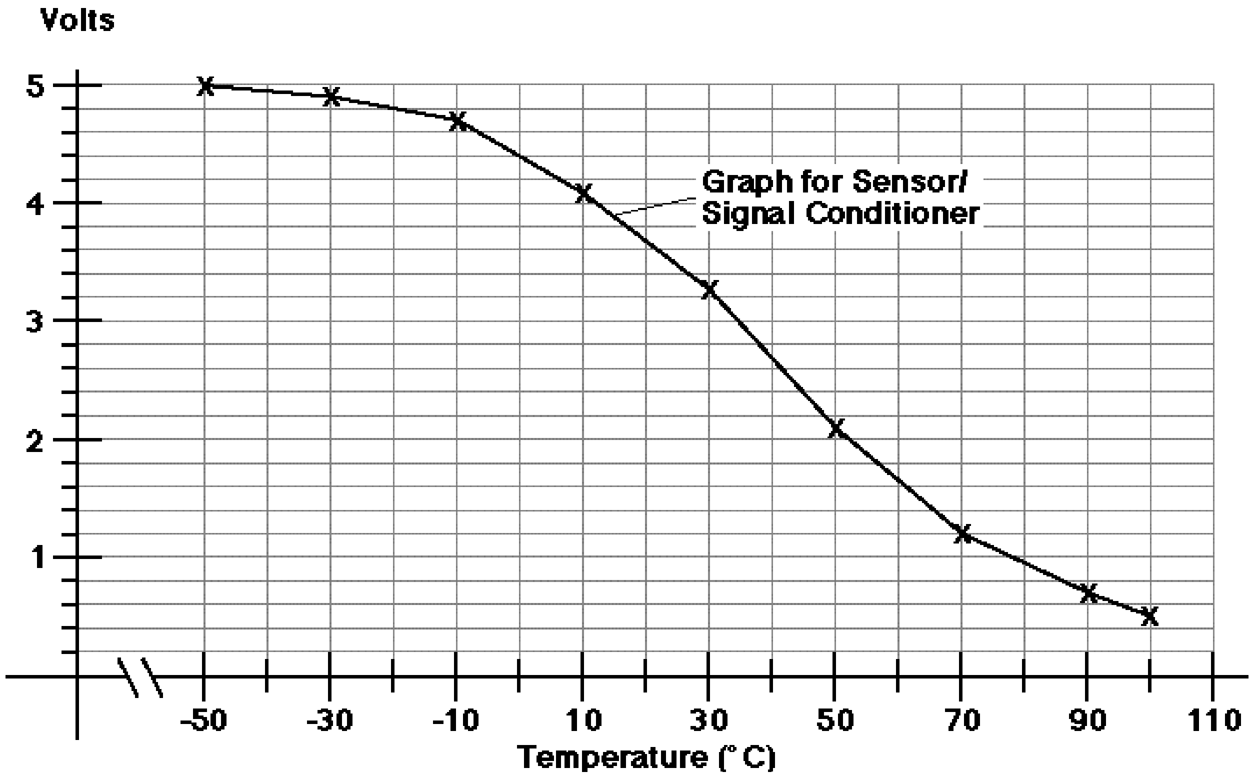
Voltage (volts)	Counts (whole numbers)
0	
1	
2	
3	
4	
5	

5) Did you use your graph or your equation to complete the chart? Explain why you chose what you did.

Section B - Temperature to Volts Equation

In this Section, you will concentrate on the sensor/signal conditioner. The graph (curve) in Figure 2 provided by the system's manufacturer shows the relationship between the temperature of the power supply and the corresponding voltage output of the sensor/signal conditioner. Even though the sensor/signal graph shows the relationship between temperature and voltage, the manufacturer did not provide an equation for the curve. In order to achieve the project goal, you must find an equation that converts temperature to volts with little error. The manufacturer recommends using the linear equation $T = 105 - 25V$ to **approximate** the curve (T = temperature, V = volts). You will now confirm that this is a good linear approximation by finding the regression line for data obtained from the curve in Figure 2 and comparing it to the manufacturer's recommended line.

Figure 2
Sensor/Signal Graph



1) Use the manufacturer’s sensor/signal graph (Figure 2) to fill in the following table. Find answers to the nearest tenth of a volt. Be as accurate in your measurements as possible.

Table 1

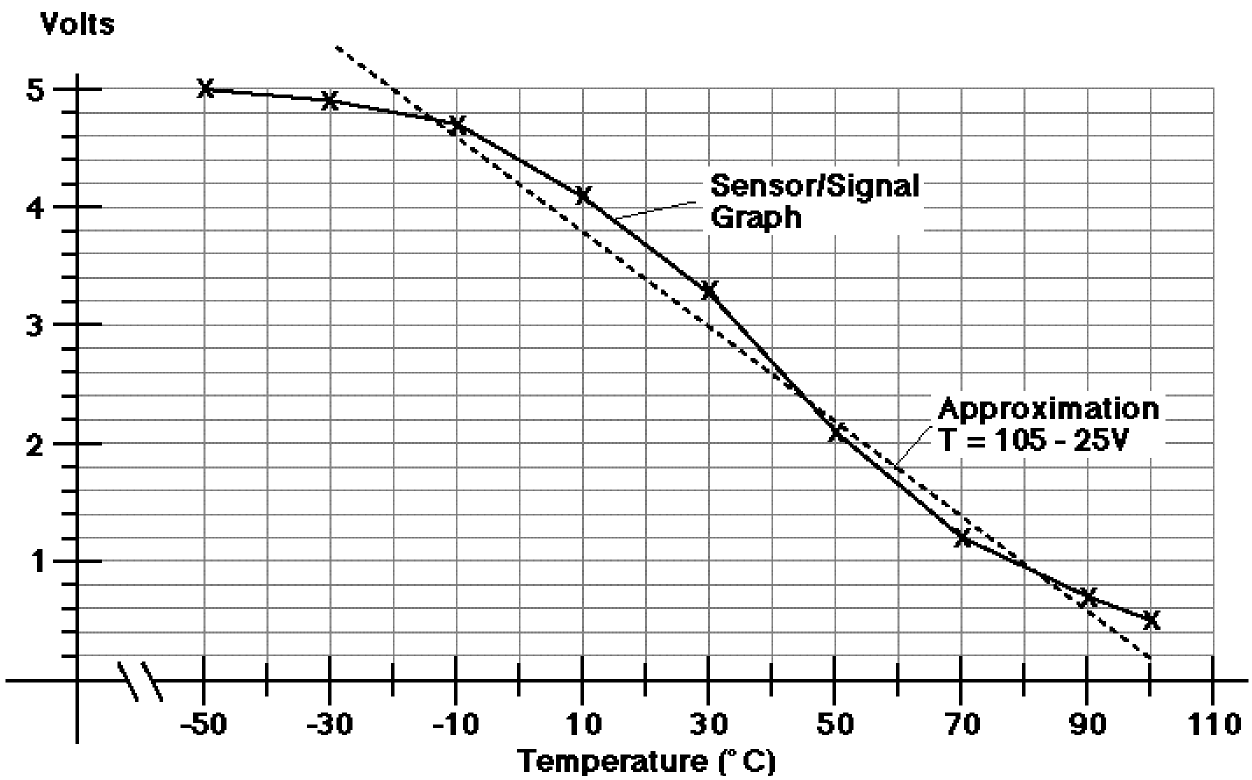
Temperature (degrees C)	Sensor (Volts)
0.0	
10.0	
20.0	
30.0	
40.0	
50.0	
60.0	
70.0	
80.0	
90.0	
100.0	

2) Why do you suppose it's reasonable to start the table at 0˚C rather than – 50˚C where the sensor/signal graph begins?

3) Find the least squares regression line for voltage in terms of temperature using the data points in Table 1. You will need to use a graphing calculator to find this best-fitting line.

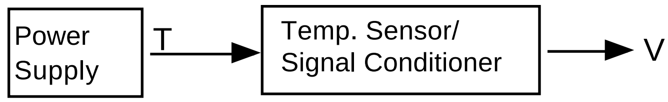
4) How does your equation compare with the equation of the manufacturer's line, T = 105 – 25V? (See Figure 3) Note that the manufacturer's equation is solved for T, so you will need to solve it for V to compare it with yours.

Figure 3
Sensor/Signal Graph with Manufacturer's Approximating Line



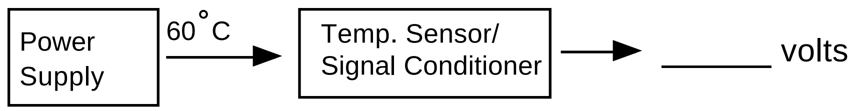
For consistency in the following work, use the manufacturer's recommended line T = 105 – 25V even though it may differ somewhat from the regression line that you found in Exercise 3.

5) Write the **manufacturer's** equation solved for V in the space below.



Equation: V = ______________________

6) Use the equation you developed in Exercise 5 to find the voltage when the temperature is 60˚C. Compare the result to the actual value of the voltage found using the Sensor/Signal curve.



7) What are the advantages and disadvantages of using (a) the line to approximate the voltage values? (b) the sensor/signal curve to approximate the voltage values?

8) With reference to Figure 3, estimate the maximum error that could occur between 0 and 100 degrees from using the manufacturer's linear approximation to the sensor/signal curve. Your error amount should be measured in volts.

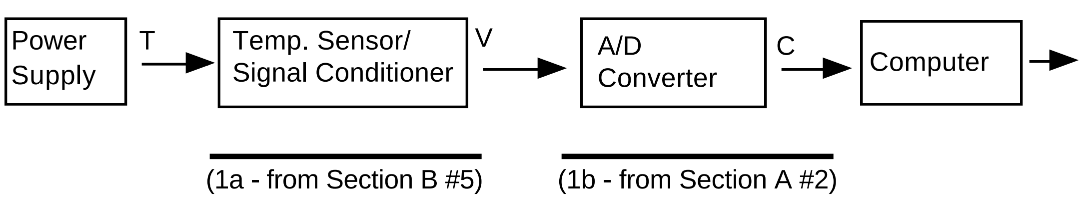
9) Looking back on your work in Section B, explain in your own words why the manufacturer suggests using the equation of a line to approximate the curve in Figure 3. Does the manufacturer's straight line give fairly accurate approximations of the voltage?

Section C - Temperature to Counts Equation and Counts to Temperature Equation

You now have a linear equation that converts the temperature of the power supply to a voltage (Section B, #5) and another linear equation that converts this voltage to an integer number of counts (Section A, #2). Write these equations in the blank spaces in Figure 4 below.

1. a,b)

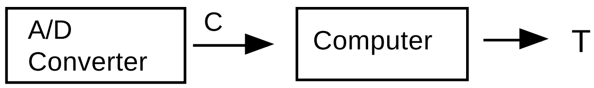
Figure 4



2) Find a single equation that converts directly from temperature to counts.

(Use the 2 equations you entered in Figure 4 to find a single equation that converts directly from temperature to counts.)

3) Finally, you need the computer to convert the integer number of counts back to a temperature reading for the computer operator. Rearrange the equation in Exercise 2 to find the equation that can be programmed into the computer to calculate the temperature given a specific number of counts. We will call this the “count to temperature equation”.



(Solve the equation of Exercise 2 for T in terms of C)

Section D - Errors from the Linear Approximation

The engineers who work on the power supply inform you that typically the power supply operates around 40˚C and will switch off at 70˚C. Check the accuracy of your results under the present system by filling in the following table.

1)

Table 2

Actual temperature of power supply (degrees C)	Voltage from sensor/signal conditioner curve recorded in Table1, Section B (Volts)	Number of counts from A/D converter (Use the equation from Section A, #2) (Round to nearest whole number)	Temperature read-out from computer (Use the equation from Section C, #3) (degrees C)	Error (Computer reading minus actual temperature) (degrees C)
30				
40				
50				
60				
70				

2) The output temperatures the computer gives do not match the actual temperature of the power supply exactly. Consider the steps involved in finding the conversion equation for the computer. At what points in the process is error introduced?

3) Around what temperatures in the interval from 30˚C to 70˚C are the approximations most accurate? Around what temperatures are they least accurate? Why could this be a problem?

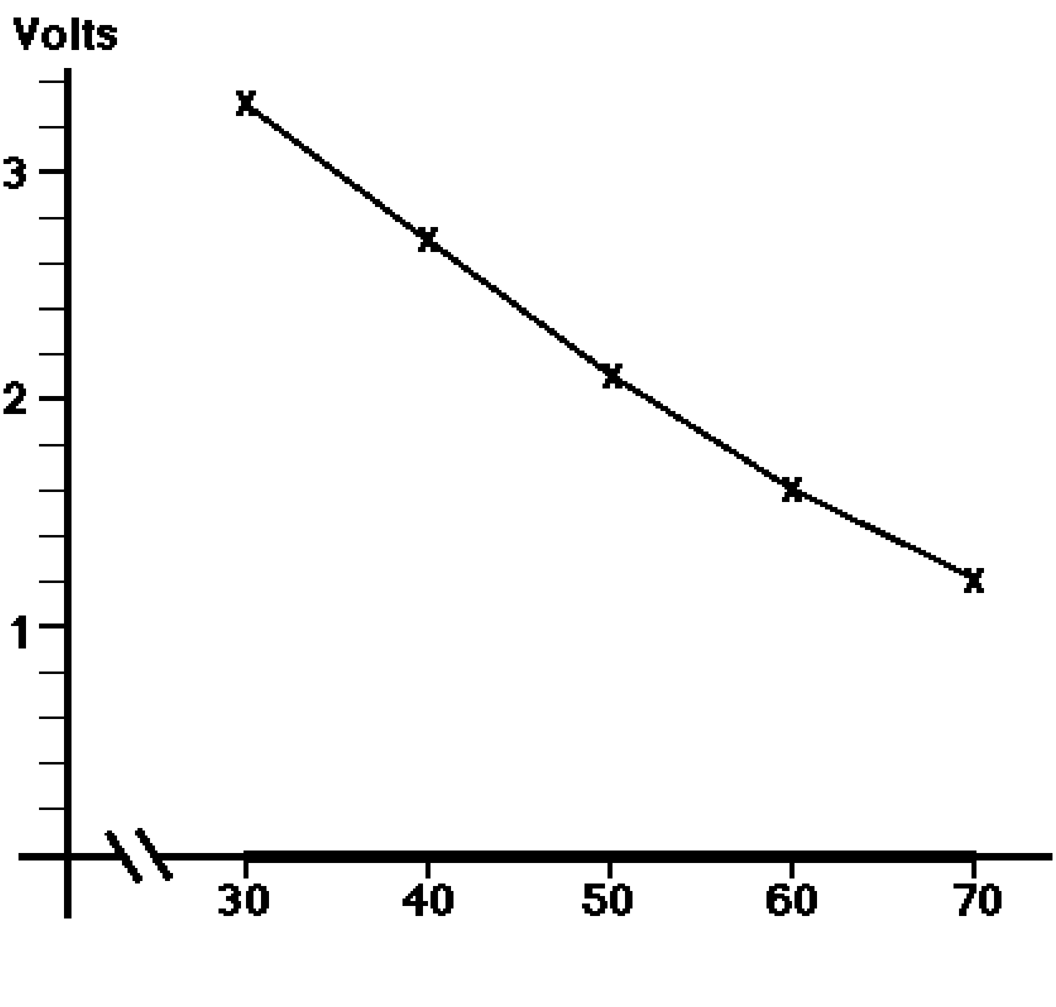
Section E - Revised Linear Approximation

Now you can see why the engineers asked you to look into this. The discrepancies between the actual temperature of the power supply and temperatures generated by the computer are unacceptably large. One way to reduce the error is to make the linear approximation to the sensor/signal curve more accurate.

Recall that the manufacturer provided the equation T = 105 – 25V for approximating voltage values over the interval from – 50˚C to 100˚C. Since the power supply operates between 30˚C and 70˚C, you decide to construct a new line that will approximate the sensor/signal curve within the restricted domain more accurately than the manufacturer's line. From this point on we will consider only the data points from the temperature interval 30˚C to 70˚C.

1) Use your values from Table 1, Part B to find the least squares regression line that approximates the sensor/signal curve restricted to the interval 30˚C to 70˚C (Figure 5). Copy this figure into your report and sketch your regression line on it.

Figure 5
Sensor/Signal Curve restricted to the temperature interval [30,70]



2) Check the accuracy of your results using your new equation by filling in the following table. (You need to find a new "count to temperature" equation for the fourth column.)

Table 3

Actual temperature of power supply (degrees C)	Actual voltage from sensor/signal conditioner curve (Volts)	Number of counts from A/D converter (rounded to nearest whole number)	Temperature that the computer will read out (degrees C)	Error (Computer reading minus actual temperature) (degrees C)
30				
40				
50				
60				
70				

3) Compare these new results to the previous results. Recall that Table 2 is associated with the manufacturer's line, and Table 3 is based on the best-fitting line that you developed by restricting the temperature to the interval 30˚C to 70˚C. What do the errors in Table 2 tell you about the accuracy of the manufacturer's approximating line compared to your line on the interval 30˚C to 70˚C? Specifically address the amount of error near 40˚C (the "normal" temperature) and the amount of error near 70˚C (the "danger zone").

4) Before you and your team take your results to your supervisors, you need to decide which equation if either to recommend. Suppose your supervisor wants the temperatures to be accurate to the nearest degree. Is the current system good enough, is your new line significantly better, or are neither good enough? You must weigh the improved accuracy against the facts that it will cost extra time, money and work to reconfigure the system. Write a persuasive argument for your supervisors on what equation you believe they should use and why.

Section F - Reducing the Error (Polynomial Regression Equations)

Your supervisor tells your group that he would prefer the temperature read-outs to be within one degree of the actual temperature. Your group decides that it may need to use a higher order polynomial to fit the sensor/signal curve better. The trouble with this is that it could be difficult or even impossible to solve the resulting equation for temperature in terms of counts. Based on the work accomplished so far, you suggest finding a set of count-temperature data pairs and then fitting this data with regression equations that have **counts as input and temperature as output.**

1) Fill in Table 4, transfer the results to Table 5, and then find linear, quadratic, cubic, and quartic regression equations that have temperature as a function of counts.

Table 4

Temperature (degrees Celsius)	Voltage (Volts)	Counts (whole numbers)
30		
40		
50		
60		
70		

Table 5

Counts (whole numbers)	Temperature (degrees Celsius)
	30
	40
	50
	60
	70

Input the count-temperature data from Table 5 into your graphing calculator to determine the following regression equations.

Linear Equation:

Quadratic Equation:

Cubic Equation:

Quartic Equation:

2) Now you must check the accuracy of your results if you adopt one of these new equations. Use the information from Exercise 1 to fill in Table 6.

Table 6

Actual temperature of power supply (degrees C)	Counts (Round to an integer)	Resulting temperature from new linear equation (degrees C)	Resulting temperature from the quadratic equation (degrees C)	Resulting temperature from the cubic equation (degrees C)	Resulting temperature from the quartic equation (degrees C)
30					
40					
50					
60					
70					

3) Before you and your team take your results to your supervisor, you need to decide which equation if any to recommend. Which equation gives the desired accuracy while requiring the least amount of calculation time? Does the extra accuracy desired by your supervisor result in improvement that is significant enough to justify the change? Keep in mind that reconfiguring the system will cost extra time, money, and work. Also many people think that computers are so fast that using a higher degree polynomial is not a big deal. However, when doing millions of calculations, a higher degree polynomial can make a substantial difference in computer time - time that we may need for other tasks.

   Write a persuasive argument for your supervisor on what equation you believe they should use and why.