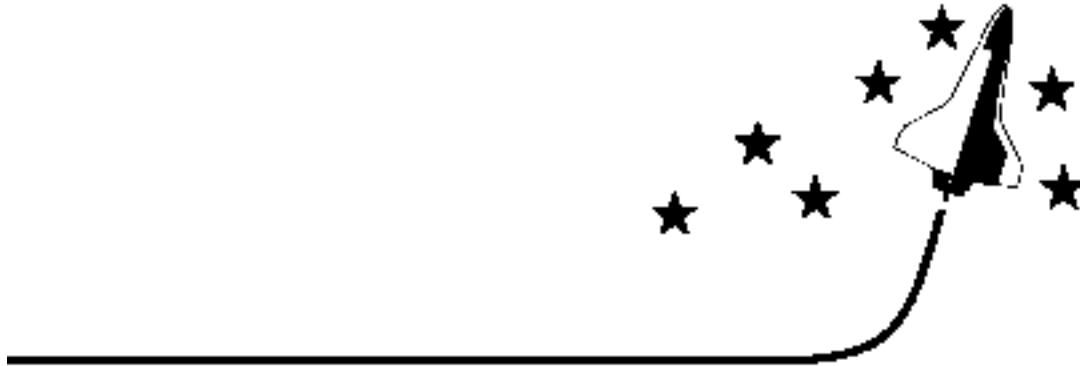


FACULTY NOTES

The LTAs and Spinoffs are designed so that each professor can implement them in a way that is consistent with his/her teaching style and course objectives. This may range from using the materials as out-of-class projects with minimal in-class guidance to doing most of the work in class. The LTAs and Spinoffs are amenable to small group cooperative work and typically benefit from the use of some learning technology. Since the objective of the LTAs and Spinoffs is to support the specific academic goals you have set for your students, the Faculty Notes are not intended to be prescriptive. The purpose of the Faculty Notes is to provide information that assists you to take full advantage of the LTAs and Spinoffs. This includes suggestions for instruction as well as answers for the exercises.



FACULTY NOTES

SPINOFF 7B

Curve Fitting for the Spacelab and Calculus III

Background Information

Topics: Graphing, graph interpretation, multivariable optimization, regression.

Level: Calculus III

Learning Objective: The students will have the opportunity to use Multivariable Calculus to fit curves that model a real life problem encountered by a NASA engineer.

Mathematical Prerequisites: Students should know three-dimensional optimization.

Technical Applications: This topic deals with commonly used electrical systems.

Time Requirement: Though it will take some perseverance, a typical Calculus III student should be able to complete this project with little or no direction from the instructor in approximately 2-4 hours depending on whether you assign the optional sections. It will go more quickly for the students if they have access to a computer algebra system to shorten up the algebraic manipulations. We feel it would be best to give this assignment and a few days later allow some class time to answer questions that the students have encountered.

Assessment: This project can be assigned as a group project or it can be done individually. We expect students at the Calculus III level to be capable of writing a self-contained, clear report. That is, we should be able to read it without referring to the original project description and understand exactly what they are doing. We encourage students to use some technology to complete the project. Of course, the regression is to be done using calculus with the calculator only as a checking device.

Solutions

Part A

- 1) $C = 204.8V$ (This should technically be a step function since the counts are integer, but it is not necessary to the problem to discuss this. The engineer left this as a continuous function.)
- 2) A linear approximation is easier to work with, gives decent approximations, and takes substantially less computing time than a higher order polynomial.
- 3) $C = 204.8(-0.04T + 4.2) = -8.192T + 860.16$
- 4) $T = -\frac{C}{8.192} + 105$

5)

Actual temperature of power supply (degrees C)	Actual voltage from sensor/signal conditioner curve (volts) (Use the graph in Figure 2)	Number of counts from A/D converter (Use the equation from Exercise 1)	Temperature that the computer will read (Use the equation from Exercise 4)
30	3.3	676	22.5
40	2.7	553	37.5
50	2.1	431	52.4
60	1.6	328	65.0
70	1.2	246	75.0

6) Clearly not.

Part B

$$1) f(a,b) = [3.3 - (30a + b)]^2 + [2.7 - (40a + b)]^2 + [2.1 - (50a + b)]^2 + [1.6 - (60a + b)]^2 + [1.2 - (70a + b)]^2$$

2) After some simplification we have the following:

$$f_a = 27,000a + 500b - 976$$

$$f_b = 500a + 10b - 21.8$$

Solving the system $f_a = 0 = f_b$, we obtain $a = -0.053$, $b = 4.83$.

Check that:

$f_{aa}(-0.053, 4.83) > 0$ and $f_{aa}(-0.053, 4.83) f_{bb}(-0.053, 4.83) - f_{ab}^2(-0.053, 4.83) > 0$. This condition for a relative minimum is well known and can be found in most multivariable calculus texts. The truth of these two inequalities means that $(-0.053, 4.83)$ is the point at which the function, $f(a,b)$ has a minimum value. Hence, the least-square regression line is $V = -0.053T + 4.83$.

$$3) C = 204.8V = 204.8(-0.053T + 4.83) = -10.8544T + 989.184$$

Solving for T in terms of C we have $T = -0.0921C + 91.13$. This equation is used to obtain the entries in the fourth column of the following table.

Actual temperature of power supply (degrees C)	Actual voltage from sensor/signal conditioner curve (volts)	Number of counts from A/D converter (use equation from Part A, #1)	Temperature that the computer will read (degrees C)
30	3.3	676	28.9
40	2.7	553	40.2
50	2.1	431	51.4
60	1.6	328	60.9
70	1.2	246	68.5