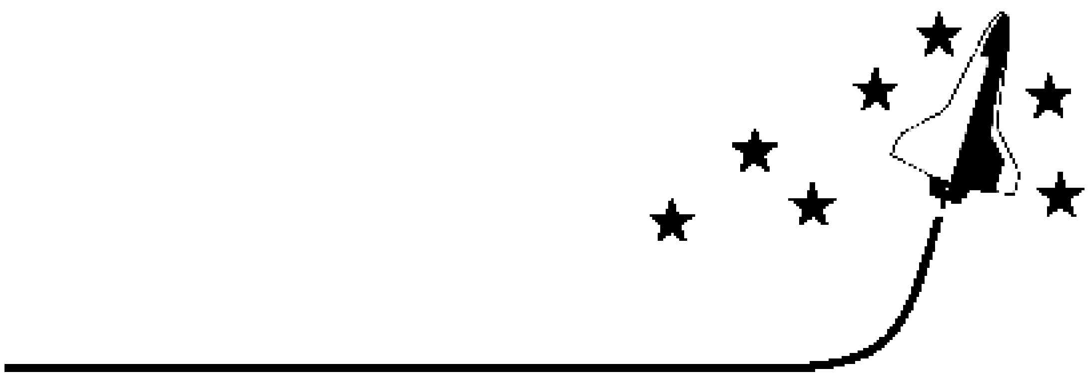
FACULTY NOTES

The LTAs and Spinoffs are designed so that each professor can implement them in a way that is consistent with his/her teaching style and course objectives. This may range from using the materials as out-of-class projects with minimal in-class guidance to doing most of the work in class. The LTAs and Spinoffs are amenable to small group cooperative work and typically benefit from the use of some learning technology. Since the objective of the LTAs and Spinoffs is to support the specific academic goals you have set for your students, the Faculty Notes are not intended to be prescriptive. The purpose of the Faculty Notes is to provide information that assists you to take full advantage of the LTAs and Spinoffs. This includes suggestions for instruction as well as answers for the exercises.



LTA 7

Houston We Have a Problem! - Curve Fitting for the Spacelab

Mathematics for
Electrical Engineering Technology
Systems EngineeringTechnology

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In 1988, Carlos Rodriguez began work for NASA at the Space Shuttle Guidance and Navigational System Branch. From 1992 to 1997 Carlos worked as a Computer Engineer at Payloads Division where he performed database coefficients calculations and computer programming for the Spacelab Checkout System. Since 1997, Carlos has worked at the Checkout Launch and Control Systems Software Applications Division. He continues to learn about the fascinating mysteries of Mathematical Models and Linear Transformations.

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FACULTY NOTES

LTA 7

Houston, We Have a Problem! - Curve Fitting for the Spacelab

Background Information

Topics: Graphing, graph interpretation, and linear equations.

Level: Intermediate Algebra, College Algebra, or Precalculus

Learning Objective: The students will have the opportunity to determine equations of lines and approximate data with linear equations using a real life problem encountered by a NASA engineer. Students will also get a brief introduction to composition of functions and inverse functions through this activity.

Mathematical Prerequisites: Students should know the basic concepts of linear equations in two variables and how to manipulate equations.

Technical Applications: This topic deals with commonly used electrical systems.

Time Requirement: We assign this as a group project. We give the students one day in class to discuss and plan strategies. We then give them 2-3 weeks outside of class to finish their work and write up a report. If this were done entirely in class and no report were required it would take about 3-4 hours of class time, if students know how to use their calculators. (Although you know how students are. They will take as much time as you give them.)

To shorten the project you could also do Sections A-D together in class and then assign Section E, the improvement of the system, as homework. This would probably take about two hours of class time with your direction.

Regardless of your approach, it would be advisable to give your students the handout the night before you plan to have them work on it in class and ask them to read and reread the first page to get a basic understanding of the project before starting in class. Be sure to remind students to bring a ruler with them if they will not be provided.

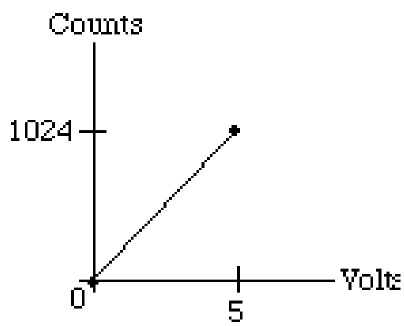
Assessment: We make this a group project and require a final report that outlines their work, answers all of the questions, and summarizes their findings. (We have purposely not left much room on the handout, so they won't be tempted to try to cram their answers into the tiny spaces provided and turn in the sheet.)

We have also done group presentations in the past and found that they work very well with this type of project. We highly recommend them if you can find class or office time in which to do them.

Solutions

Section A - Volts to Counts Equation

1)



Technically, this should be a step function since the counts are integer, but it is not necessary to the problem (or probably even desirable at this level) to discuss this. By the way, the engineer who did this problem also left it as a continuous function.

2) $C = 204.8V$

3) The students can show that the point (2.5, 512) is a solution to the equation, they can show that the slope between the three points is constant, or if their graph is very accurate they can show that it is a point on the line.

4)

Voltage (volts)	Counts (whole numbers)
0	0
1	205
2	410
3	614
4	819
5	1024

(Remind students to round their answers to the nearest whole number!)

5) Most students choose the equation since it is easier for them to use and more accurate than trying to estimate the voltages from the graph. Some students choose to graph the equation on their calculator and use the table, trace, or value features.

Section B - Temperature to Volts Equation

1) The sensor/signal curve is used to fill in the following table.

Table 1

Temperature (degrees C)	Sensor (Volts)
0.0	4.4
10.0	4.1
20.0	3.7
30.0	3.3
40.0	2.7
50.0	2.1
60.0	1.6
70.0	1.2
80.0	1.0
90.0	0.7
100.0	0.5

2) It's reasonable to start the graph at 0 degrees since it is highly unlikely that the system will be operating below a freezing temperature. In addition, deleting the negative values of temperature will make the linear approximation better within the temperature range that the power supply operates.

3) $V = 4.41 – 0.04227T$

4) Solving the regression equation for T gives approximately $T = 104 – 24V$, and the equation of the manufacturer's line is $T = 105 - 25V$. Another way to make the comparison is to solve the manufacturer's equation for V to obtain $V = 4.2 – 0.04T$ and to note that the linear regression equation is $V = 4.41 – 0.04227T$.

5) $V = 4.2 - 0.04T$

6) $V(60) = 1.8$ volts. The actual value from the curve is approximately 1.6 volts, a difference of 0.2 volts.

7. a) Using the line is easier than having to estimate values from the graph. Additionally, the linear equation provides a very close approximation to the voltage.
   b) An advantage to using the sensor/signal curve rather than a straight line approximation is that the sensor signal curve, within the precision limits of the sensor/signal device and within each individual's perceptual ability to read the graph, provides exact values of the voltage rather than approximations to them.

8) The error appears to never be more than approximately 0.3 volts.

9) The linear approximation seems to give fairly good estimates.

Section C - Temperature to Counts Equation and Counts to Temperature Equation

1. a) $V = -0.04T + 4.2$ (or $V = -T/25 + 105/25$)
   b) $C = 204.8V$

2) $C = 204.8(-0.04T + 4.2) = -8.192T + 860.16$

3) $T = -C/8.192 + 105$

Section D - Errors from the Linear Approximation

1) Be sure to take some time to clearly explain this chart. The students are asked to see how well their equation converts the actual number of counts back into temperature. Remind them that the actual values of the voltage should be entered in the second column and should match the values in the corresponding rows of Table 1. Also, remind the students that since their "volts to counts" equation from Section A was exact, it is O.K. to use that equation to fill in the counts column where the results are rounded to the nearest whole number. Finally, the students can use their conversion equation (counts-to-temperature) to see how well it approximates the process.

Table 2

Actual temperature of power supply (degrees C)	Voltage from sensor/signal conditioner curve recorded in Table 1, Section B (Volts)	Number of counts from A/D converter (Use the equation from Section A, #2) (Round to the nearest whole number)	Temperature reading from computer (Use the equation from Section C, #3) (degrees C)	Error (Computer reading minus actual temperature) (degrees C)
30	3.3	676	22.5	-7.5
40	2.7	553	37.5	-2.5
50	2.1	431	52.4	2.4
60	1.6	328	65.0	5.0
70	1.2	246	75.0	5.0

2) Some error is introduced when the sensor/signal curve is approximated by a line. A small amount of error is introduced when the continuous measurement of voltage is converted into an integer number of counts at the A/D converter.

3) The best approximations seem to occur around 40 to 50 degrees and the worst approximations seem to occur around 30 degrees. Also, the error near 70 degrees could be a problem because 70 is in the "danger zone". Fortunately the temperature reading is too high, and hence the computer will not be damaged by running at too hot a temperature. On the other hand, the power supply will cut off when it does not need to, which may waste valuable time.

Section E - Revised Linear Approximation

1) Use a graphing calculator to determine the regression line that best fits the data in the first 2 columns of Table 1 in Section B. These are repeated for your convenience below.
Using a TI-83™, the regression the line that best fits the 6 data points is $V = -0.053T + 4.83$.

Actual temperature of power supply (degrees C)	Actual voltage from sensor/signal conditioner curve (You will have to measure these off of the graph in Section B) (Volts)
30	3.3
40	2.7
50	2.1
60	1.6
70	1.2

2) Note that the first three columns in the chart below should remain the same as before and the final two columns should show improvement. The students should also note not only whether the amount of error has improved, but also whether the temperature that the computer is reading as 70 degrees is too high or too low. If it is reading too low it could be a problem since it reaches the "critical" zone before we are aware.

For Column 3, use $C = 204.8V$ from Section A Exercise 2
For Column 4, form the composite of the the two functions, $C = 204.8V$ and $V = -0.053T + 4.83$ to express C in terms of T. Solve this equation for T terms of C.

$C = 204.8V = 204.8(-0.053T + 4.83) = -10.8544T + 989.184$ and thus
$T \approx -0.0921C + 91.13$

Table 3

Actual temperature of power supply (degrees C)	Actual voltage from sensor/signal conditioner curve (Volts)	Number of counts from A/D converter (rounded to nearest whole number)	Temperature that the computer will read out (degrees C)	Error (Computer reading minus actual temperature) (degrees C)
30	3.3	676	28.9	-1.1
40	2.7	553	40.2	0.2
50	2.1	431	51.4	1.4
60	1.6	328	60.9	0.9
70	1.2	246	68.5	-1.5

3) The following table compares the errors for the manufacturer's line with those for your line.
Note: The new equation reads too low at 70˚.

Actual temperature of power supply (degrees C)	Error based on manufacturer's given equation (Computer reading minus actual temperature) (degrees C)	Error based on new equation (Computer reading minus actual temperature) (degrees C)
30	-7.5	-1.1
40	-2.5	0.2
50	2.4	1.4
60	5.0	0.9
70	5.0	-1.5

4) Answers will vary. Credit might be weighted to take into account how well students support their arguments.

Section F - Reducing the Error (Polynomial Regression Equations)

1) linear: $T \approx -0.0916C + 90.9113$

quadratic: $T \approx 5.6053 \times 10^{-5} C^2 - 0.1433C + 101.5043$

cubic: $T \approx -1.9790 \times 10^{-7} C^3 + 3.2968 \times 10^{-4} C^2 - 0.2617C + 117.3586$

quartic: $T \approx 7.8135 \times 10^{-11} C^4 - 3.4062 \times 10^{-7} C^3 + 4.2344 \times 10^{-4} C^2 - 0.2878C + 119.9592$

(Note the slight difference in the linear equation in this exercise compared to the linear equation in Section E, Exercise 2, due to the different approaches.)

Table 4

Temperature (degrees Celsius)	Voltage (volts)	Counts
30	3.3	676
40	2.7	553
50	2.1	431
60	1.6	328
70	1.2	246

Table 5

Counts	Temperature (degrees Celsius)
676	30
553	40
431	50
328	60
246	70

2)

Table 6

Actual temperature of power supply (degrees C)	Counts (Rounded to nearest whole number)	Resulting temperature from new linear equation (degrees C)	Resulting temperature from the quadratic equation (degrees C)	Resulting temperature from the cubic equation (degrees C)	Resulting temperature from the quartic equation (degrees C)
30	676	28.99	30.25	29.97	30.002
40	553	40.26	39.40	39.99	40.002
50	431	51.43	50.15	49.96	50.001
60	328	60.87	60.53	60.00	60.001
70	246	68.38	69.65	69.98	70.001

3) If your boss wants correct temperatures when rounded to the nearest degree, then the cubic would be the best choice. While the quartic appears highly accurate, the cubic has the advantage of requiring less computation time, a high enough degree of accuracy, and it actually overestimates the temperature at the critical point of 70˚ which might be safer. It should be noted that the linear approximation gives decent results also. If your boss could tolerate slightly more error, this could save a lot of calculation time.