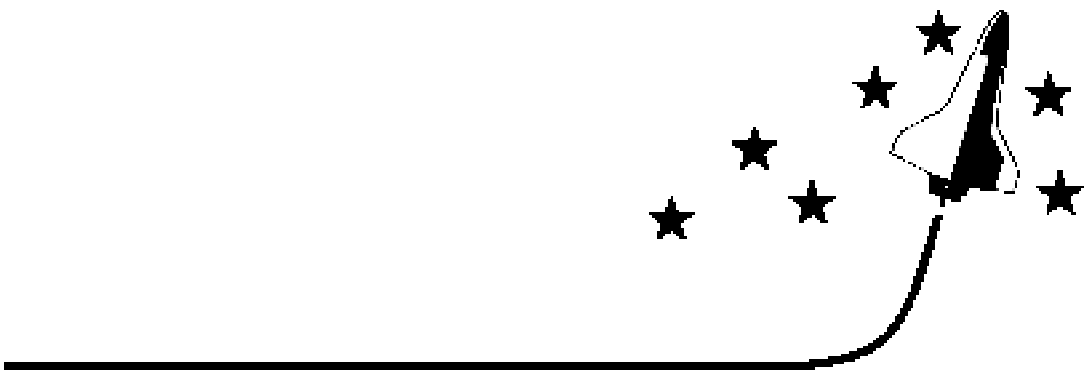
FACULTY NOTES

The LTAs and Spinoffs are designed so that each professor can implement them in a way that is consistent with his/her teaching style and course objectives. This may range from using the materials as out-of-class projects with minimal in-class guidance to doing most of the work in class. The LTAs and Spinoffs are amenable to small group cooperative work and typically benefit from the use of some learning technology. Since the objective of the LTAs and Spinoffs is to support the specific academic goals you have set for your students, the Faculty Notes are not intended to be prescriptive. The purpose of the Faculty Notes is to provide information that assists you to take full advantage of the LTAs and Spinoffs. This includes suggestions for instruction as well as answers for the exercises.



FACULTY NOTES

LTA 4

Just Say "NO" to Cracks in the Space Shuttle

Background on the Kennedy Space Center

The 140,000-acre John F. Kennedy Space Center (KSC) is the major launch site for the National Aeronautics and Space Administration (NASA).

Prior to the Space Shuttle era, manned launches from KSC included the Apollo missions, which landed men on the Moon; three flights to Skylab, the first United States Space Station; and the Apollo/Soyuz Test Project Mission.

NASA also launched a wide variety of unmanned spacecraft using expendable rockets, lifting off from Cape Canaveral in Florida and from Vandenberg Air Force Base in California. These have included weather and communications satellites, orbiting scientific satellites, lunar explorers and landers, and interplanetary probes to Mercury, Venus, Mars, Jupiter, Saturn, Uranus and Neptune.

Except for operational areas, the KSC reservation is designated as a national wildlife refuge.

Background on the Space Shuttle

A Space Shuttle consists of an Orbiter, an external tank, and two solid rocket boosters (SRBs). The Shuttle is 184.2 feet in height with a gross liftoff weight of about 4,500,000 pounds and a total liftoff thrust of 7,700,000 pounds. This thrust decreases during the two minutes the solids burn to about 3,000,000 pounds during the last 10 seconds. After the SRBs are discarded, the Orbiter engines burn for 6 ½ more minutes and take the vehicle to near-orbital velocity.

The delta-winged Orbiter resembles an airplane and is about the size of a DC-9 jetliner. The four Orbiters in service are Columbia, Discovery, Atlantis, and Endeavour. Only the Orbiters have names.

A Space Shuttle Orbiter is 122.17 feet in length with a wingspan of 78.06 feet. The three main engines each produce 375,00 pounds of thrust at sea level and 470,000 pounds in the vacuum of space. The cargo bay is 60 feet in length and 15 feet in diameter. The Space Shuttle could launch a bus into orbit.

After the boosters separate from the Shuttle, they descend by parachute to a watery landing. They are retrieved by two specially-designed ships and returned to Kennedy Space Center for refurbishment and eventual reuse.

After the Orbiter's three main engines shut down, the external tank is jettisoned, breaks up in the atmosphere, and falls into the Indian Ocean. It is the only major Shuttle component that is not reused.

The Orbiter uses its smaller maneuvering system engines, supplied by on-board propellants, to enter orbit. After performing its mission, the Orbiter uses these engines again to slow the vehicle and return through the atmosphere to land as an unpowered glider.

More than 200 significant modifications have been made to the Orbiter fleet since 1986. One of these was a crew escape system. It allows the crew members to bail out if the Orbiter is in level flight within the atmosphere, but cannot reach a safe runway.

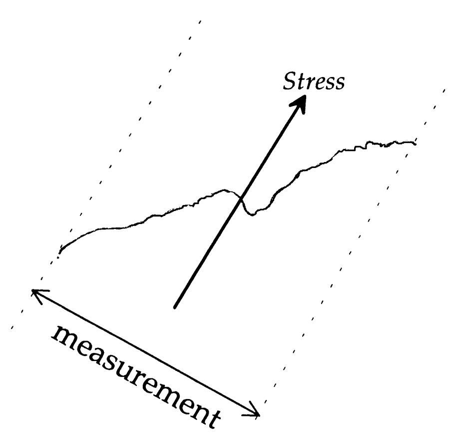
The Space Shuttle will be the primary launch vehicle for the United States for years to come.

Background on Fracture Mechanics

The field of Fracture Mechanics has only existed since approximately the time of World War II. Naval ships were manufactured in the U.S. and then shipped for use in the North Sea. At least one of these ships actually snapped in half as a result of cracks and the reaction metal has to extreme cold. Once the cracks reached the critical stage they literally grew at the speed of sound and 'snapped' the ship in half.

The equations and scenarios given in this LTA are a simplification of what Fracture Mechanical Engineers really use and experience. For example, deciding what the **allowable** K (K_a), Stress Intensity Factor should be is an extensive process. The **critical** K (K_c) must first be determined, and it is not as simple as looking it up on a chart. There are many factors which contribute to K_c. Initially, you consider the type of material used (for example high strength steel) and then other factors such as temperature are considered. Once a K_c is determined, then the **allowable** Stress Intensity Factor K_a is calculated. This is not simply a matter of some sort of ratio or rule of thumb, but the material's end use must be considered. If the material is used for the Space Station, then you want a big buffer between **critical** and **allowable**. On the other hand, if the material is for a less demanding application or if it can be readily repaired, the allowable can push the envelope more.

Another item which has been simplified is that of measuring the length of a crack. In the LTA it is expected that students will simply measure the crack from end to end. Actually the crack should be measured *normal* to the direction of stress which caused the crack, as indicated below:



Curricular Information

General

It is expected that this will be a culminating activity after students have solved simple equations with **square roots**. Students should also have had experience with **dimensional analysis, basic graphing, and solving literal equations**.

Depending upon a college's curriculum, students at the **Intermediate Algebra** level should be capable of completing this activity. However, because of an increase in maturity level, it might be better suited for students at the **College Algebra** level. It is best done in groups and therefore suggested that **the students have some previous experience in group work.**

The activity is designed to increase the student's ability to work with multiple equations, a variety of unfamiliar units, and with technology.

Assessment

Although assessment will vary with the instructor, assessment techniques could include the following:

- Require an oral presentation on part of the project or on the whole project.
- Require a written report for the project, a part of the project, or one of the exercises.
- Give students a problem similar to the project and have them write a recommendation to the project director about whether or not corrective action is needed.
- On exams be sure to include problems where students encounter small/large numbers and use formulas that are not already solved for the appropriate variable.
- Expect students to write or give an oral report on the relationships between the variables in this unit, based on what they have seen numerically and graphically in the problems.

Group Work

Group work can be incorporated in many ways; here are a few suggestions:

- Allow students to work on the exercises together.
- Have students work together on the project and present a group report.
- Have different groups assigned to different problems in the project and then have each group do a presentation about the portion they solved.
- Make sure students have already worked in groups before this activity.

Technological Information

This activity was written to be calculator-independent. However, it is expected that all students will have access to a scientific calculator at a minimum. Ideally, students will be able to use computer algebra software such as Derive™ or a graphics calculator such as the TI-83™ (this is especially advisable for parts 3 through 5 in **The Project** section). With the domain of $0 \le a \le 0.5$, once students determine the appropriate range, the graph's characteristic features are obvious. Students are then easily able to determine the needed values of K. Since we are dealing with a combination of very large and very small numbers, students need to use the increased accuracy of a calculator that can store or carry answers. When using two equations, students need to use the calculator's results from the first equation (not rounded) for the second equation.

Timeline for Classroom Use

This activity is intended for approximately two class periods with students doing some work outside of class. The following is one suggestion as to how to handle the workload. This is in no way the only choice since everyone has different schedules and demands on classroom time.

Class period 1 — Allow students to become familiar with the problem, physics background and formulas. Work some of the exercises in class and assign the rest as homework.

Class period 2 — One week later (to allow for this to all sink in), have students work in groups to do the project. If time allows have them do oral presentations or assign a written report to be handed in at a later date.

Solutions

Your Job

1) **Pressure:** If you increase the pressure of the gas, the crack will be more likely to rupture the pipe.

2) **Thickness of pipe wall:** If you decrease the thickness of the pipe wall, the crack will be more likely to rupture the pipe.

3) **Radius of pipe:** If you increase the radius of the pipe but keep the wall thickness constant, the crack will be more likely to rupture the pipe.

4) **Stress on pipe:** The vulnerability of a pipe to being ruptured by a crack, referred to as stress on the pipe, increases if the pressure is increased, increases if the wall thickness is decreased, and increases if the radius of the pipe is increased.

5) **Length of crack:** If you increase the length of the crack, the crack will be more likely to rupture the pipe.

6) **Stress Intensity Factor:** The likelihood that a crack will rupture a pipe, referred to as the stress intensity factor, increases if the length of the crack is increased and increases if the stress on the pipe is increased.

Exercises

Note: The length of the crack in the tank measures approximately 1 inch. That is, $2a \approx 1$ inch.

1) Length of crack = 28 mm so a = 14 mm
s = 112 MPa
$K = s\sqrt{p\,a} = 112\,\text{MPa}\sqrt{p \cdot 14\,\text{mm}} = 743\ \text{MPa - mm}^{1/2}$

2) Crack 1: a = 0.1 in.; P = 50,000 psi; K = 28,025 psi- in$^{1/2}$
Crack 2 a = 0.05 in. P = 100,000 psi; K = 39,633 psi- in$^{1/2}$

3) $P = 600$ psi, $r = 12 \text{ ft} \cdot \frac{12 \text{ in}}{1 \text{ ft}} = 144$ in. Set up a table using $s = \frac{600 \cdot 144}{t}$.

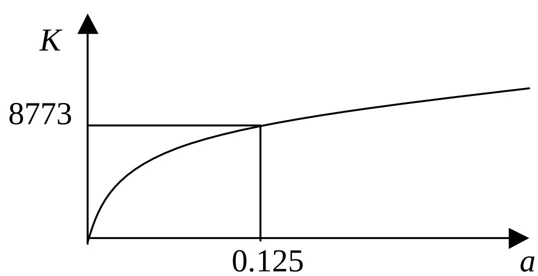
The table below uses TblMin = 0 and ΔTbl = 0.1, where X = t and $Y_1 = s$.

X	Y_1	
0	ERROR	
.1	864000	
.2	432000	
.3	288000	
.4	216000	
.5	172800	
.6	144000	

X=0

s varies inversely with t. As t increases, s decreases.

4) $K < 14{,}000 \text{ psi} \sqrt{p \cdot (0.125 \text{ in})} = 8{,}773 \text{ psi - in}^{1/2}$. The idea is that any crack we cannot detect must produce a value of K that is allowable. The allowable values of K correspond to the half-crack lengths from 0 to 0.125 as shown in the graph of K vs. a.



Back to the Problem

Criterion 1

We base our decision on the given allowable value of K (K-value). Note that for the average radius to be 48 in, the 8 ft dimension given in Figure 1 must be considered an average diameter.

$$K_a = 100{,}000 \text{ psi - in}^{1/2},\ t = 0.375 \text{ in},\ P = 110 \text{ psi},\ r = 48 \text{ in},\ a = 0.5 \text{ in}$$

$$s = \frac{(110)(48)}{0.375} = 14{,}080 \text{ psi}$$

$$K = 14{,}080\sqrt{p \cdot 0.5} = 17647 \text{ psi - in}^{1/2}$$

Since 17,647 < 100,000 the crack is safe and requires no action.

Criterion 2

As an alternative approach, we compare the length of the crack with the thickness of the tank. Length of crack in tank = $2a \approx 1\text{ in} > 2\cdot\left(\frac{3}{8}\right)\text{ in} = \frac{3}{4}\text{in}$. Therefore, by this criterion the crack is unsafe and it would require corrective action.

The Lamé Equation

5) $r_o = 2.375\text{ in}$; $r_i = 2.07\text{ in}$; average radius = 2.2225 in; wall thickness = 0.305 in;

$$\frac{\text{radius}}{\text{wall thickness}} = 7.287 > 6$$

$P = 6000$ psi

Thin Wall: $s = \dfrac{6{,}000\text{ psi}\cdot 2.2225\text{ in}}{0.305\text{ in}} = 43{,}721\text{ psi}$

Lamé: $s = 6{,}000\text{ psi}\cdot\dfrac{(2.375\text{ in})^2 + (2.07\text{ in})^2}{(2.375\text{ in})^2 - (2.07\text{ in})^2} = 43{,}927\text{ psi}$

6) average radius = 35.5 in, wall thickness = 1 in

$$\frac{\text{radius}}{\text{wall thickness}} = 35.5 > 6$$

$$120{,}000\text{ psi} = \frac{P\cdot 35.5\text{ in}}{1\text{ in}} \Rightarrow P = 3{,}380\text{ psi}$$

7) $r = 30$ mm; $s = 100$ MPa
$P = 10$ MPa

$$100\text{ MPa} = \frac{10\text{ MPa}\cdot 30\text{ mm}}{t} \Rightarrow t = 3\text{ mm}$$

$$250\text{ MPa} = \frac{10\text{ MPa}\cdot 30\text{ mm}}{t} \Rightarrow t = 1.2\text{ mm}$$

t (mm)	**s (MPa)**
0.1	3,000
0.2	1,500
0.3	1,000
0.4	750
0.5	600
0.6	500

8) $a = 0.125$ in; P = 600 psi; $t = 1.5$ in; $r_o = 24$ in $\quad r = \dfrac{24 \text{ in} + 22.5 \text{ in}}{2} = 23.25 \text{ in}$

$$s = \frac{(600\,\text{psi})(23.25\text{ in})}{1.5\text{ in}} = 9{,}300\text{ psi}$$

$K = 9{,}300\,\text{psi} \cdot \sqrt{p \cdot 0.125\text{ in}} = 5{,}828\text{ psi - in}^{1/2}$. It is safe to operate.

To find the highest allowable stress, use $K_{\max} = s_{\max}\sqrt{p a}$.

$$s_{max} = \frac{75{,}000\,\text{psi - in}^{1/2}}{\sqrt{p \cdot 0.125\text{ in}}} = 119{,}683\text{ psi}$$

$$s_{\max} = \frac{P_{\max} \cdot r}{t}$$

$$P_{\max} = \frac{119{,}683\text{ psi} \cdot 1.5\text{ in}}{23.25\text{ in}} = 7{,}721\text{ psi}$$

The highest allowable pressure for the pipe is approximately 7,721 psi.

The Project

1) $r_o = 6.1$ feet = 73.2 inches
$P = 900$ psi
$s = 130{,}000$ psi

$r = \dfrac{r_o + r_i}{2}$ and $t = r_o - r_i \Rightarrow r_i = r_o - t$

$$r = \frac{2r_o - t}{2}$$

$$130{,}000\,\text{psi} = \frac{900\text{ psi} \cdot \dfrac{(2)(73.2\text{ in}) - t}{2}}{t} \Rightarrow 130{,}000\,t = 65{,}880 - 450\,t$$

$$130{,}450\,t = 65{,}880 \Rightarrow t = 0.505\text{ in}$$

NOTE: The use of r_o as r results in a thickness of 0.507 in rather than 0.505 in.

2) $a = 5/32$ in
$r_o = 6.1$ feet = 73.2 in
$r_i = 72.7$ in

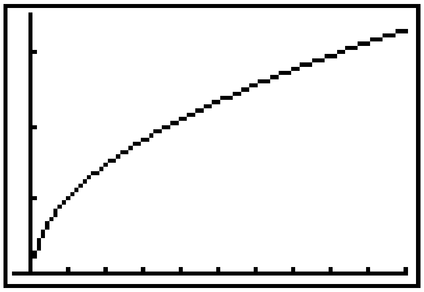
$$s = 900\,\text{psi}\left(\frac{(73.2\text{ in})^2 + (72.7\text{ in})^2}{(73.2\text{ in})^2 - (72.7\text{ in})^2}\right) \Rightarrow s = 131{,}312\text{ psi}$$

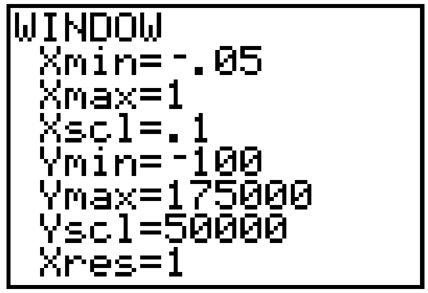
$$K = 131{,}312\text{ psi} \cdot \sqrt{p \cdot \left(\frac{5}{32}\text{ in}\right)} = 92{,}000\text{ psi - in}^{1/2};$$

$92{,}000\text{ psi - in}^{1/2} <$ Maximum Allowable K of $100{,}000\text{ psi - in}^{1/2}$.

Thus, no action is needed.

3) Let x = length of the crack, then $a = \frac{x}{2}$. Graph $K = y = 131{,}312\sqrt{p \cdot \frac{x}{2}}$.





4) For a 0.1 inch crack, K = 52,043 psi- $\text{in}^{1/2}$.
For a 0.25 inch crack, K = 82,288 psi- $\text{in}^{1/2}$.
For a 0.3 inch crack, K = 90,142 psi- $\text{in}^{1/2}$.

5) The bold entries in the table show those K values that indicate needed corrective action for the associated cracks. $K = y = 131{,}312\sqrt{p \cdot \frac{x}{2}}$

Crack Length (inches)	**K** (psi- $\text{in}^{1/2}$)
0	0
0.05	36,800
0.1	52,043
0.15	63,740
0.2	73,600
0.25	82,288
0.3	**90,142**
0.35	**97,364**
0.4	**104,086**
0.45	**110,400**
0.5	**116,372**