SPINOFFS

Spinoffs are relatively short learning modules inspired by the LTAs. They can be easily implemented to support student learning in courses ranging from prealgebra through calculus. The Spinoffs typically give students an opportunity to use mathematics in a real world context.

LTA - SPINOFF 3A — A Cost-Benefit Analysis of the Doppler Radar Wind Profiler Project at the Kennedy Space Center

LTA - SPINOFF 3B — The Lognormal Distribution: A Teaching Note Based on an Analysis of Wind Change at the Kennedy Space Center

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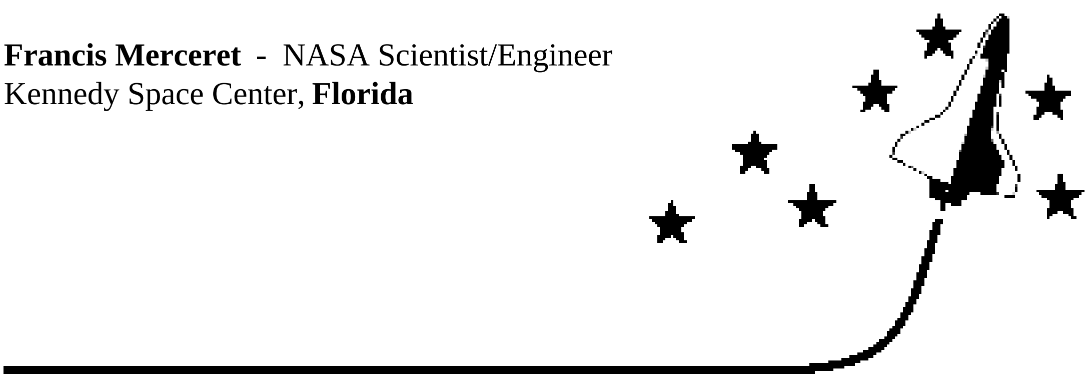

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SPINOFF 3B

The Lognormal Distribution: A Teaching Note Based on an Analysis of Wind Change at the Kennedy Space Center

Solutions

1. a) The normal and uniform distributions are symmetric about a vertical line drawn through their mean, and therefore they are not skewed to the left or right. On the other hand, the right tail of the lognormal distribution is much longer than its left tail. Thus, the lognormal distribution is skewed to the right.

   b) All three distributions have the same mean and standard deviation. Since the lognormal's peak is greater than the peak of the normal distribution, the lognormal is leptokurtic. In contrast, the uniform distribution's peak is less than that of the normal distribution. Thus, the uniform distribution is platykurtic.

2)

$\sigma^2 > 0 \quad e^{\sigma^2} > 1 \quad \omega > 1$. This implies that:

$\omega^4 > 1$, $2\omega^3 > 2$, and $3\omega^2 > 3 \quad \omega^4 + 2\omega^3 + 3\omega^2 > 6$

Thus, $\omega^4 + 2\omega^3 + 3\omega^2 - 3 > 3$.

Hence, Kurtosis > 3. You may also prove the result graphically.

3) As σ increases, the mean of the lognormal distribution increases.
As σ increases, the median of the lognormal distribution remains the same.
As σ increases, the mode of the lognormal distribution decreases.
As σ increases, the standard deviation of the lognormal distribution increases.
As σ increases, the lognormal distribution becomes much more skewed to the right.
As σ increases, the kurtosis of the lognormal distribution greatly increases.

4) For $\mu = 0$ and $\sigma = 1.5$:
Mean = 3.0802
Median = 1
Mode = 0.1054
Std. Dev. = 8.9738
Skewness = 33.468
Kurtosis = 10078.25

5) For $\mu = 1$ and $\sigma = 0.5$:
Mean =3.0802
Median = 2.7183
Mode = 2.1170
Std. Dev. = 1.6416
Skewness = 1.7501
Kurtosis = 8.8984

6) **Case 1:** $W = \Lambda(0.2, 0.7^2)$

Note that $\mu_W = e^{(0.2+0.49/2)} = e^{0.445} = 1.5605$

$$\sigma_W^{\ 2} = e^{2(0.2)+0.49}\left[e^{0.49} - 1\right] = 1.5398$$
$$\sigma_W = 1.2409$$

Then:

$$
\begin{aligned}
& P(W > \mu_W + 3\sigma_W) \\
& = P(W > 1.5605 + 3 \ 1.2409) \\
& = P(W > 5.2832) \\
& = P(e^Y > 5.2832) \\
& = P(Y > \ln(5.2832)) \\
& = P(Y > 1.6645) \\
& = P(Z > (1.6645 - 0.2)/0.7) \\
& = P(Z > 2.0921428) \\
& = 0.0182128
\end{aligned}
$$

Case 2: W is normally distributed with the mean and standard deviation equal to 1.5605 and 1.2409 respectively.

If W is normally distributed, the probability of a 3-Sigma event is independent of the mean and standard deviation.

$$P(W > \mu_W + 3\sigma_W) = P\left(Z > \frac{(\mu + 3\sigma) - \mu}{\sigma}\right) = P(Z > 3) = 0.00135$$

The ratio of 3-Sigma probability for lognormal to 3-Sigma probability for normal is equal to $\frac{0.0182}{0.00135} = 13.48$. Thus the 3-Sigma event is about 13.5 times more likely for the lognormal distribution than for the Gaussian (normal) distribution.

7) On your TI-83™ graphing calculator, let $Y_1 = \frac{1}{\sqrt{2\pi}(0.7)x} e^{-0.5\left(\frac{\ln(x)-0.2}{0.7}\right)^2}$. Graph the function on your calculator, and use the CALC menu to evaluate the definite integral:

$$\int_0^{5.2832} \frac{1}{\sqrt{2\pi}(0.7)x} e^{-0.5\left(\frac{\ln(x)-0.2}{0.7}\right)^2} dx = 0.98178906$$

Thus, P(W>5.2831) = 1 – 0.98123443 = 0.01821094

8) Use the program, LOGNZ as follows:

At the MU prompt enter 0.2, at the SIGMA prompt enter 0.7, and at the N prompt enter 3.
The lognormal 3-Sigma probability = 0.0182117.
The normal 3-Sigma probability = 0.00135.
The ratio is 13.49.

9) The following table is based on $\mu = 1.5$ and $\sigma = 0.60$.

	N=1	N=2	N=3	N=4	N=5	N =6
Lognormal	0.1265	0.0446	0.0171	0.0071	0.0032	0.0015
Normal	0.1587	0.0228	0.0013	3.1686e-5	2.871e-7	9.9012e-10
Ratio	0.7975	1.9584	12.6897	225.31	11107.75	1525934.94

10) The answers will vary depending on the 30 samples that the computer selects from the lognormal distribution, $\Lambda(1,\ 0.5^2)$.