SPINOFFS

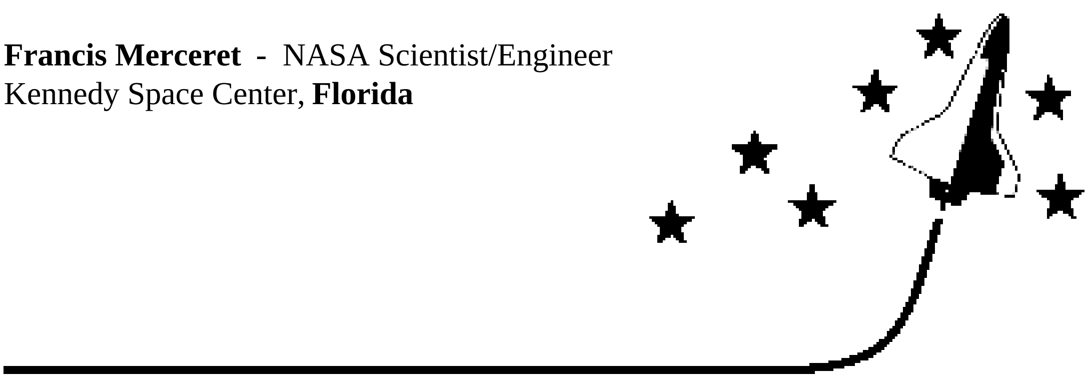
Spinoffs are relatively short learning modules inspired by the LTAs. They can be easily implemented to support student learning in courses ranging from prealgebra through calculus. The Spinoffs typically give students an opportunity to use mathematics in a real world context.

LTA - SPINOFF 3A	A Cost-Benefit Analysis of the Doppler Radar Wind Profiler Project at the Kennedy Space Center
LTA - SPINOFF 3B	The Lognormal Distribution: A Teaching Note Based on an Analysis of Wind Change at the Kennedy Space Center

Dennis Ebersole - AMATYC Writing Team Member
Northampton Community College, Bethlehem, **Pennsylvania**

Brian Smith - AMATYC Writing Team Member
Dawson College, Montreal, Quebec, **Canada**
(Currently at McGill University, Canada)

Francis Merceret - NASA Scientist/Engineer
Kennedy Space Center, **Florida**



Project Grant Team

John S. Pazdar
Project Director
Capital Comm-Tech College
Hartford, Connecticut

Patricia L. Hirschy
Principal Investigator
Asnuntuck Comm-Tech College
Enfield, Connecticut

Peter A. Wursthorn
Principal Investigator
Capital Comm-Tech College
Hartford, Connecticut

This project was supported, in part, by the
National Science Foundation
Opinions expressed are those of the authors
and not necessarily those of the Foundation

SPINOFF 3A

A Cost-Benefit Analysis of the Doppler Radar Wind Profiler at the Kennedy Space Center

Problem Statement

The Doppler Radar Wind Profiler (DRWP) at the Kennedy Space Center (KSC) can detect sudden wind velocity changes that would not be noticed using the traditional method of determining wind velocity changes -- a weather balloon sent aloft one hour before launch. The cost of operating the DRWP is $250,000 per year. With NASA funding being limited, the scientists at KSC need to know whether this expense can be justified. Your mission, should you decide to accept it, is to compare the benefits of using the DRWP with its annual operating cost ($250,000) and to make a recommendation to either stop using the DRWP or to continue funding it. You must justify your recommendation. The DRWP is used for approximately eight Shuttle launches per year and 2 Titan rocket launches per year. Shuttles have a crew of 6 persons while Titan rockets are unmanned.

Mathematical Aside

The benefit associated with the DRWP depends on the value of the Shuttle, the value of a Titan rocket, and the value of the payloads they are carrying. Another factor is the probability that the DRWP will detect a sudden wind velocity change not detectable by traditional means, thus avoiding the destruction of a launch vehicle and its payload. To determine the benefit associated with the DRWP, we need to understand the concept of mathematical expectation. We first introduce several terms.

An **experiment** is the observation of an occurrence. For example, the Engineers at KSC conducted an experiment when they used the DRWP to record changes in wind velocities over a long period of time. A **sample space** is the set of all possible outcomes. For the purposes of the KSC experiment the possible outcomes were the possible changes in wind velocity from the last reading made by the weather balloon, so our sample space would be the set of all possible changes in wind velocity. An **event** is a subset of the sample space. The KSC scientists were interested in the event: “changes in wind velocity with absolute value greater than 20 m/s”, a subset of all the possible outcomes.

Example 1: Consider the following experiment. You will toss a coin and a six-sided die. You are interested in obtaining a head on the coin and either a 5 or 6 on the die. Find the sample space and event.

Solution:

Let S be the name for the sample space. Then

$S = \{(H,1), (H,2), (H,3), (H,4), (H,5), (H,6), (T,1), (T,2), (T,3), (T,4), (T,5), (T,6)\}$

The event E we are interested in is:

$E = \{(H,5), (H,6)\}$

The **probability** of an event is a measure of the likelihood that the event will occur. In most instances we use **experimental probability**, obtaining the probability by looking at a large number of data. For example, the insurance industry uses data from across the United States to determine the probability that someone fitting a certain profile (e.g., 18 year old, unmarried male) will have an automobile accident involving medical expenses. To find the experimental probability *p* we divide the number (m) of times an event occurs by the number (n) of times the experiment was repeated.

$$p = \frac{m}{n}$$ Experimental Probability

In some cases we can use logical reasoning to determine the **theoretical probability** of an event. If all outcomes are equally likely (often not the case), the theoretical probability can be found by dividing the number of outcomes S in the event by the total number of outcomes T in the sample space. The outcomes in the event of interest are often referred to as successes.

$$\mathrm{p} = \frac{\mathrm{S}}{\mathrm{T}}$$

Exercises:

1) If a coin and a die are tossed simultaneously, find the theoretical probability that the coin shows heads and the die shows a 5 or a 6.

2) Three fair coins are tossed. List the sample space for this experiment. Consider the event "exactly two heads". List all outcomes which satisfy this event and specify the theoretical probability of the event.

3) The program for the TI-83™ shown below simulates tossing two dice and finding the sum of the two dice. Run the program 100 times and use the results to find the experimental probability of getting either a 7 or an 11 for the sum.

```
PROGRAM:DICE
:randInt(1,6)+randInt(1,6)→S
:Disp S
```

Mathematicians use the concept of **mathematical expectation** or **expected value** to indicate the value of an event with different "winnings" and probabilities for each possible outcome. To find the expected value of an event, you must multiply the “winnings” associated with each outcome by the probability of that outcome. The sum of these products is the expected value of the event. In symbols, the expected value V of an event composed of outcomes that have probabilities $p_1, p_2, p_3, \ldots$ and winnings $w_1, w_2, w_3, \ldots$ is given by the following formula:

$$V = w_1 \; p_1 + w_2 \; p_2 + w_3 \; p_3 + \ldots$$

Example 2: A fair coin and fair die are tossed. If a head and a 1 or 2 shows, you win \$2. If a head and a 3 or 4 shows, you win \$5. If a head and a 5 or 6 show, you win \$10. Otherwise, you lose \$5. What is the mathematical expectation for this game?

Solution: Let p_1 be the probability of getting a head and a 1 or 2, p_2 be the probability of getting a head and 3 or 4, p_3 be the probability of getting a head and a 5 or 6, and p_4 be the probability of getting any other outcome. Then $p_1 = p_2 = p_3 = \frac{2}{12}$. Also, $p_4 = \frac{6}{12}$. The associated winnings are $w_1 = 2, w_2 = 5, w_3 = 10, w_4 = -5$. The expected value V is

$$\begin{aligned} V &= \frac{2}{12}\; 2 + \frac{2}{12}\; 5 + \frac{2}{12}\; 10 + \frac{6}{12}\; (-5) \\ &= \frac{4}{12} + \frac{10}{12} + \frac{20}{12} + \frac{-30}{12} \\ &= \frac{4}{12} \approx \$0.33 \end{aligned}$$

If you play this game repeatedly, you can expect to win an average of \$0.33 per game.

We now have the mathematical tools needed to solve the problem described in the beginning paragraph of this Spinoff. The following four-step model for problem solving will help to guide you through the solution process. Try to remember these five steps, for they can be applied to almost any problem.

1) **Define the problem**
   Refer to the problem statement at the beginning of this investigation. Discuss the problem in your team. What information do you need if you are going to be able to make a recommendation to the Kennedy Space Center's Director? Make a list of the information needed and present the list to your instructor who has collected the information you will need.

   What assumptions must you make in order to complete your mission?

   State the problem in your own words.

2) **Devise a plan**
State in your own words how you will determine a dollar value for the benefit of using the DRWP.

3) **Carry out the plan**
Determine the expected value of the DRWP using your assumptions. Use this number to decide whether or not to recommend continued use of the DRWP.

4) **Look back**
There may be loss of life if the Shuttle is lost. How would this factor affect your recommendation to the KSC Director?