LTA 3

NASA - AMATYC - NSF
Project Coalition

Kennedy Space Center

The Doppler Radar Wind Profiler Project:
Vector Analysis of Wind Changes Affecting
Shuttle Launch at the Kennedy Space Center

Mathematics for Engineering Technology

Aeronautical
Space

Capital Community-Technical College

At a remote site on KSC, a team conducts weather research that could influence Space Shuttle launch commit criteria.

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The Doppler Radar Wind Profiler Project: Vector Analysis of Wind Changes Affecting Shuttle Launch at the Kennedy Space Center

Mathematics for
Aeronautical Engineering Technology
Space Engineering Technology

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Background

Space vehicles are preprogrammed to fly a certain trajectory in order to reach the correct orbit. This programming takes into account the forces of the winds expected to be encountered by the vehicle on ascent. If the winds actually encountered differ from those anticipated, the forces on the vehicle (called "loads") will differ. This could result either in the inability of the guidance system to fly the required trajectory, or in excessive loads leading to breakup of the vehicle. Either consequence is unacceptable. NASA/KSC will not launch a spacecraft if there is a perceived danger to the vehicle as a result of wind conditions.

Questions:

1) What sort of wind conditions may endanger a launch?

2) Suggest possible changes in wind conditions over a short period of time which may endanger a launch.

Description of the DRWP

The Kennedy Space Center (KSC) Doppler Radar Wind Profiler (DRWP) is an instrument used to measure wind speed and direction as a function of height. It provides a measurement every 150 meters of altitude above the two-acre antenna. You can think of several parallel zones of air, one above the other, each 150 meters wide. Each zone is called a "gate" from radar terminology in which distance along the radar beam is measured in "range gates". The lowest gate, referred to as gate 1, is at an altitude of 2011 meters, and there are 112 gates. A "wind profile" is a complete set of data from all 112 gates. That is, a wind profile provides the data needed to determine the velocity (speed and direction) of the wind in all gates at a certain instant of time. These data may be plotted as graphs of wind speed and direction versus height.

The wind velocity in each plane has two perpendicular components that are especially important: one component lies in the vertical plane determined by the Shuttle's launch trajectory; the other is perpendicular to the plane of the launch trajectory. These are referred to respectively as the in-plane and out-of-plane wind velocity. For a Shuttle launch, the profile collected at $t - 4$ hours (four hours before scheduled launch) is used to create the guidance program. The profiles collected at $t - 2$ hours and $t - 1$ hour are then compared with the $t - 4$ profile. If the absolute value of the change in either the in-plane or out-of-plane wind velocity relative to the $t - 4$ reading is greater than a predetermined amount (typically in the order of 20 m/s), the launch may be delayed or scrubbed. In deciding whether to delay or to scrub a flight, it is important to realize that for each mission there is a window of time within which the shuttle can be launched. Also, it takes several hours to recalculate the guidance program on the basis of the data taken at $t - 1$ hour. The launch will be delayed if it is possible to alter the guidance program within the launch window; otherwise it will be scrubbed.

Questions:

3) If 10:00 a.m. is the scheduled launch time, t, when is the $t - 4$ wind profile taken? When is the $t - 1$ wind profile taken? If the launch were delayed for 3 hours, what is the new scheduled launch time?

4) The scheduled launch time for a shuttle is 10:00 a.m. on Monday. The launch window ends at 11:30 a.m. on the same day. The in-plane wind velocity at $t - 4$ hours was 37 m/s and at $t - 1$ hour it was 15 m/s. The absolute value of the difference of these in-plane velocities is 22 m/s. Since this exceeds 20 m/s, the flight must be delayed or scrubbed. Which will it be, delayed or scrubbed? Justify your answer. (Assume that it takes 4 hours to recalculate the guidance program)

The Mathematical Model

Introduction To Vector Algebra In 2-Dimensions (Mathematical Aside)

Wind velocity has two dimensions - the speed of the wind and its direction. We cannot fully describe the wind velocity with a single number such as 7 m/s (a scalar quantity), since this does not tell us the direction of the wind. We need to know both the speed and the direction in order to compensate for wind velocity during a launch. Explain why both the wind speed and direction are important.

Single numbers such as 7 m/s are called **scalars**. We need a mathematical construct that provides two (or more) pieces of information simultaneously. One such construct is the **vector**. We can think of a vector as a directed line segment. The two vectors in Figure 1 are NOT equal, even though the two segments are the same length. Why?

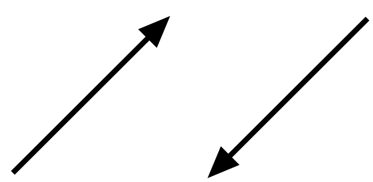

Figure 1

In order to perform operations on vectors it is useful to superimpose a coordinate system. A two dimensional coordinate system is sufficient because the wind velocities addressed in this project are confined to planes. Since the positioning of the coordinate system is arbitrary, we may locate the initial point of the directed line segment (vector) anywhere on the coordinate system without changing the vector. The two vectors in Figure 2 ARE equal. Why?

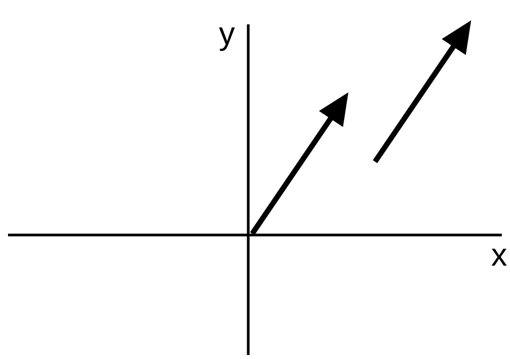

Figure 2

Since placement of the initial point does not affect the vector, we often use the origin as the initial point to simplify computations. We can then define a vector in one of two ways. One obvious choice is to give the speed r (the length or magnitude of the vector) and the direction θ (the angle that the vector makes with the positive x-axis, measured counterclockwise). Refer to Figure 3.

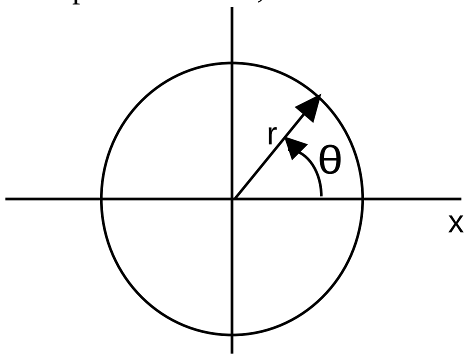

Figure 3

We think of the terminal point of the vector as lying on a circle of radius *r* centered at the origin. Any vector is completely determined by the radius of the circle on which its terminal point lies and the angle from the positive x-axis to the vector. This is called a **polar coordinate system**. The notation for a vector in polar coordinates is < r, θ >. The vector < 3, 120˚ > is shown in Figure 4.

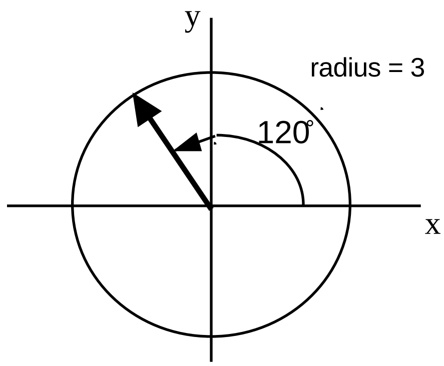

Figure 4

Questions:

5) Use polar coordinate notation to write the vector in Figure 5.

6) Draw the vector < 2, 225˚ > in Figure 6.

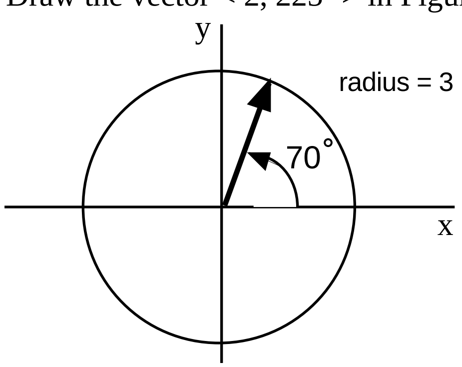

Figure 5

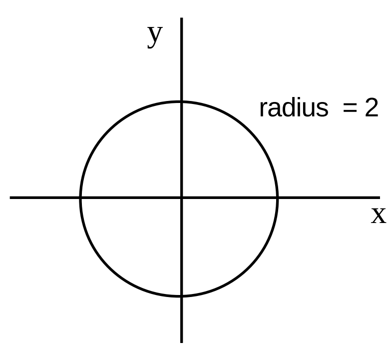

Figure 6

We can also define a vector by using the rectangular coordinates (x, y) of the terminal point of the vector. The notation < x, y > is used to represent a vector in a rectangular coordinate system. The vector < – 4, 3 > is shown in Figure 7. Sketch the vector < 3, –2 > on the coordinate system in Figure 8.

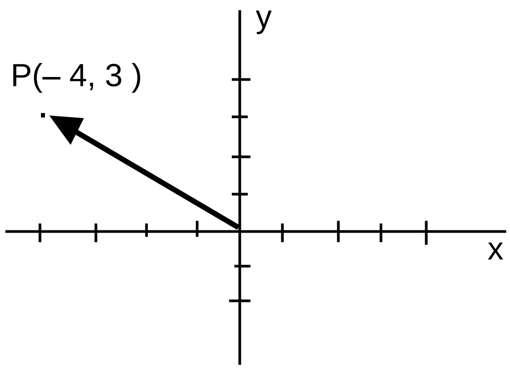

Figure 7

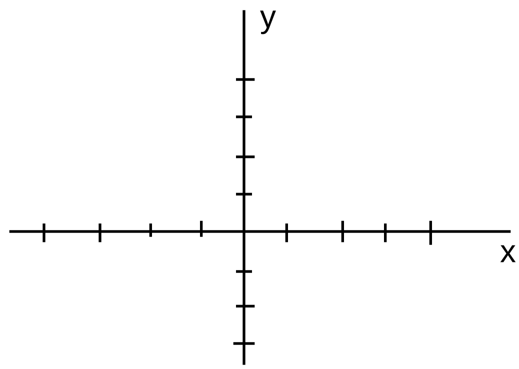

Figure 8

We will now develop some formulas for converting between the two representations of a vector. These formulas will be based on the definitions of the trigonometric functions: sine, cosine, and tangent, and on the Pythagorean Theorem. Figure 9 should help you to see how a vector's polar coordinates, r and θ, are related to its rectangular coordinates, x and y.

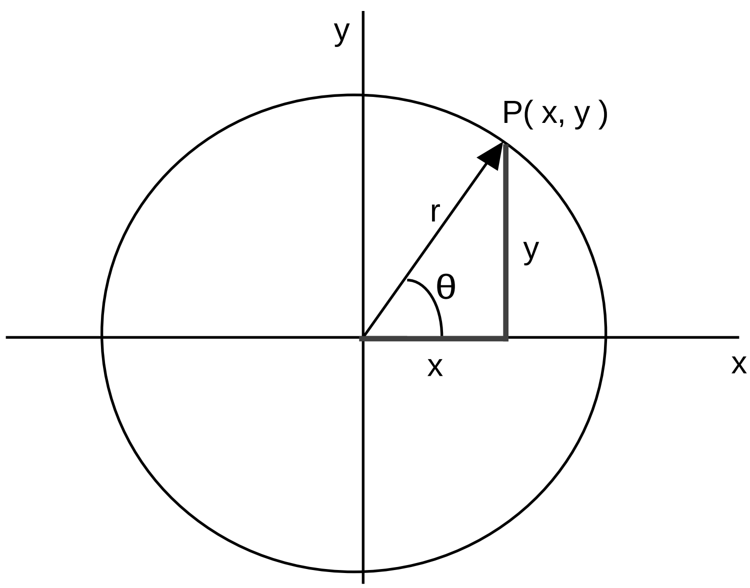

Figure 9

Polar-to-Rectangular Conversions

$a)\quad \frac{x}{r} = \cos(\theta) \qquad x = r\cos(\theta)$

$b)\quad \frac{y}{r} = \sin(\theta) \qquad y = r\sin(\theta)$

Example 1: The vector < 12, 150˚ > can be expressed in rectangular form as follows.

$$x = 12\cos(150^\circ) = -10.39$$
$$y = 12\sin(150^\circ) = 6$$

Thus, in rectangular form the vector is < – 10.39, 6 >

Rectangular-to-Polar Conversions

$a)\quad x^2 + y^2 = r^2 \qquad r = \sqrt{x^2 + y^2}$

$$b)\quad \tan(\theta) = \frac{y}{x} \qquad \left\{ \begin{array}{ll} \theta = \tan^{-1}\left(\frac{y}{x}\right) & \text{if } x > 0 \text{ and } y \geq 0 \\ \theta = \tan^{-1}\left(\frac{y}{x}\right) + 180^\circ & \text{if } x < 0 \text{ and } y \geq 0 \\ \theta = \tan^{-1}\left(\frac{y}{x}\right) + 180^\circ & \text{if } x < 0 \text{ and } y < 0 \\ \theta = \tan^{-1}\left(\frac{y}{x}\right) + 360^\circ & \text{if } x > 0 \text{ and } y < 0 \end{array} \right\} \quad \text{where} \quad 0^\circ \leq \theta < 360^\circ$$

$c)$ If $x = 0$ and $y > 0$, then $\theta = 90^\circ$

If $x = 0$ and $y < 0$, then $\theta = 270^\circ$

If you choose any point P(x,y) in the coordinate plane, the rectangular-to-polar conversion formulas will give the degree measure of the angle between the positive x-axis and the vector drawn from the origin to P. The measure of θ will be from 0˚ up to 360˚.

Example 2: The vector < – 4, 3 > can be expressed in polar form as follows.

$$r = \sqrt{(-4)^2 + 3^2} = 5$$

$$\theta = \tan^{-1}\left(\frac{3}{-4}\right) + 180^\circ = -36.9^\circ + 180^\circ = 143.1^\circ$$

Notice that we had to add 180˚ in order to get an angle in quadrant II, because x = – 4 < 0 and y = 3 > 0 places the vector in quadrant II.
Thus, in polar form the vector is < 5, 143.1˚ >.

Exercises:

Sketch each of the following vectors on a coordinate system and convert them to their alternative forms.

1) < 8, 75˚ >

2) < 42, 215˚ >

3) < 2, 8 >

4) < – 16, – 22 >

Technology can be used to quickly convert between rectangular and polar coordinates. The following programs make the conversions on Texas Instruments calculators. (The programs shown are for a TI-83™. You may have to make minor alterations to execute it on a different graphing calculator.) Notice that U and V are used in the programs instead of x and y to emphasize that the results represent DRWP data.

Program 1: Conversion from polar coordinates to rectangular coordinates

```
PROGRAM:VECT1
:ClrHome
:Degree
:Fix 2
:Disp "ENTER R"
:Prompt R
:Disp "ENTER THETA"
:Prompt T
:R*cos(T) → U
:R*sin(T) → V
:ClrHome
:Disp "U:"
:Disp U
:Disp "V:"
:Disp V
```

Example 3: Convert the polar vector < 19, 210˚ > to a rectangular vector, < u, v >. The following TI-83™ screens show the input and output for converting this vector to rectangular form.

```
ENTER R
R=?19
ENTER THETA
T=?210
```

Polar Input R = 19, T = 210˚

```
U:
        –16.45
V:
         –9.50
          Done
```

Rectangular Output U = – 16.45, V = – 9.50

Exercises:

5) Convert the polar vector < 3, 120˚ > to rectangular form using the appropriate formulas, and match your steps with the instructions in Program 1.

6) Give an example of a conversion from polar to rectangular form where the vector is in the fourth quadrant.

Program 2: Conversion from rectangular coordinates to polar coordinates

```
PROGRAM:VECT2
:ClrHome
:Degree
:Fix 2
:Disp "ENTER U"
:Prompt U
:Disp "ENTER V"
:Prompt V
:√(U^2+V^2)→R
:tan^-1(V/U)→T
:If U<0 and V>0
:Then
:T+180→T
:Goto 10
:Stop
:End
:If U>0 and V<0
:Then
:T+360→T
:Goto 10
:Stop
:End
:If U<0 and V<0
:Then
:T+180→T
:Goto 10
:Stop
:End
:Lbl 10
:ClrHome
:Disp "R:"
:Disp R
:Disp "THETA"
:Disp T
```

Example 4: Convert the rectangular vector < 10, 5 > to polar form and confirm your results using Program 2.

$$r = \sqrt{10^2 + 5^2} = \sqrt{125} = 11.18$$

$$\theta = \tan^{-1}\left(\frac{5}{10}\right) = 26.57^\circ$$

The following TI-83™ screens show the input and output for converting the rectangular vector < 10, 5 > to polar form with output $r = 11.18$ and $\theta = 26.57^\circ$.

```
ENTER U
U=?10
ENTER V
V=?5
```

Rectangular Input U = 10, V = 5

```
R:
        11.18
THETA:
                 26.57
         Done
```

Polar Output $r = 11.18$, $\theta = 26.57^\circ$

Exercises:

7) Convert the rectangular vector < – 8, 6 > to polar form using the appropriate formulas, and match your steps with the instructions in Program 2.

8) Explain why you added 180° to the inverse tangent of the quotient y divided by x in Exercise 7.

9) Give an example of a conversion where you would need to add 360° to the inverse tangent. Explain each step of the program.

Addition of Vectors

It is very easy to add two rectangular vectors. We can visualize the **resultant vector S,** the vector obtained by adding two vectors, as shown in Figure 10. If vector **A** represents the first wind velocity and vector **B** represents the second wind velocity, the combined effect of these two velocities is the single wind velocity represented by the vector **S**. We can find the resultant vector geometrically by first constructing a parallelogram with adjacent sides **A** and **B**. The resultant vector is obtained by drawing the diagonal of the parallelogram from the origin to the opposite vertex.

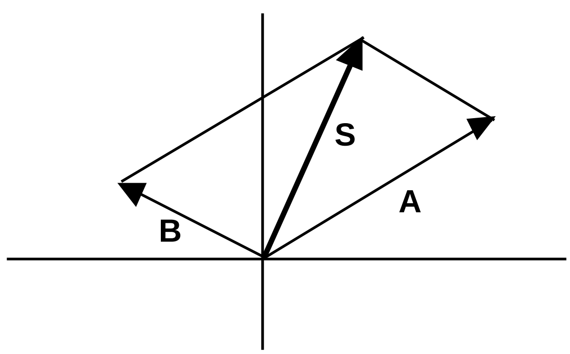

Figure 10

Example 5: Graph the vectors **A** and **B** and sketch a single vector **S** that is equivalent to these two vectors. (We call this resultant vector the sum of **A** and **B**, written **A+B**.)

a) **A** = < 3, 2 > , **B** = < 4, – 3 >

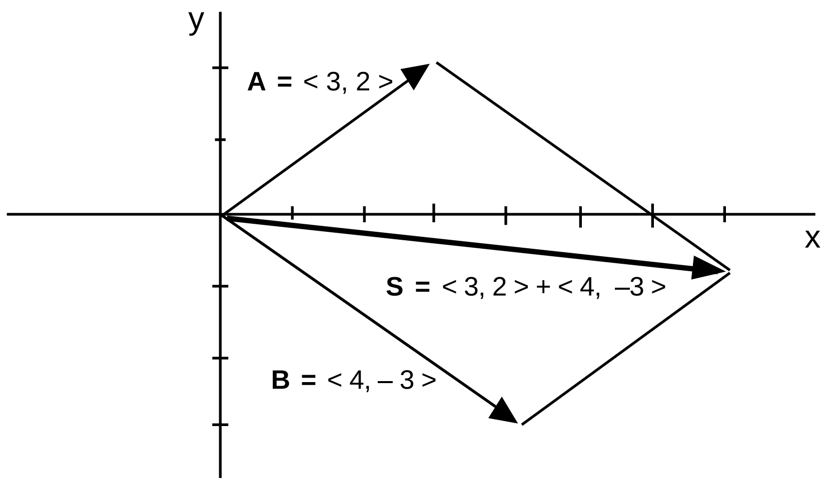

Figure 11

b) **A** = < 6, 4 > , **B** = < – 3, 2 >

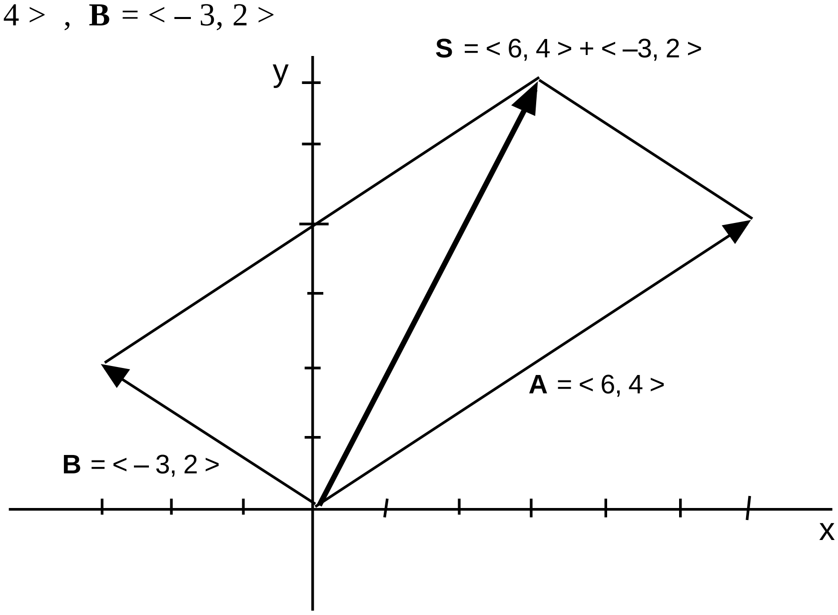

Figure 12

Question:

7) Look at the resultant vectors in Figures 11 and 12. Write each resultant vector in the form < *x*, *y* >. How could you find the resultant vector without graphing each vector? Explain the reasoning you used.
< 3, 2 > + < 4, – 3 > = ___________

< 6, 4 > + < – 3, 2 > = ___________

Example 6: Draw a graph to determine the sum **S** of the vectors < – 4, – 5 > and < 3, – 2 >. Use your graphing calculator to check your result.

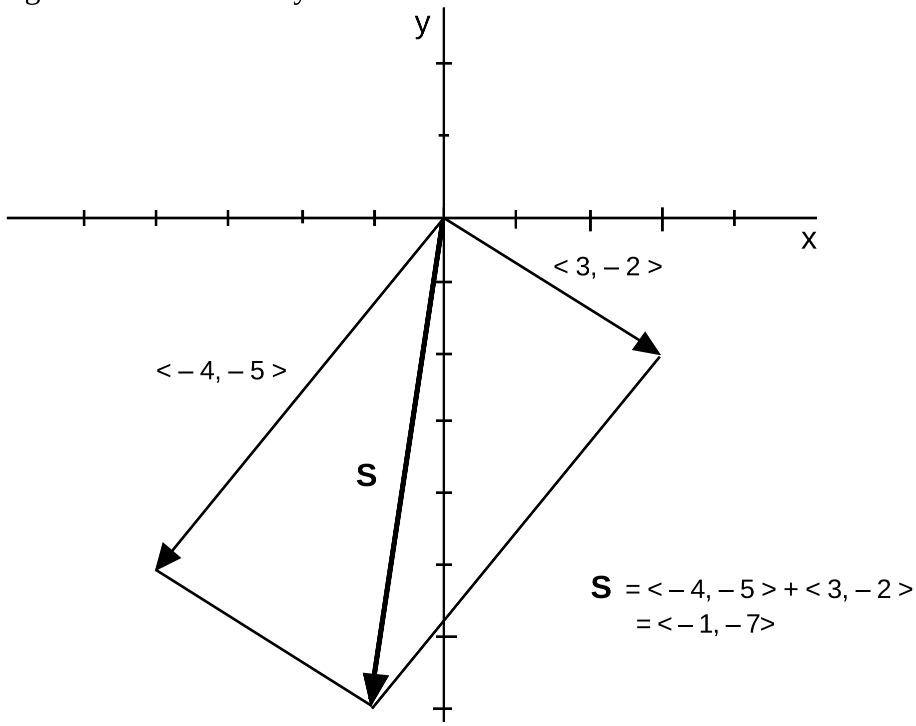

Figure 13

On the TI-82/83™ and TI-85/86™ calculators a vector is represented using double square brackets [[and]]. Thus, the vector < – 4, – 5 > will be represented as [[– 4,– 5]] and the vector < 3, – 2 > will appear as [[3,– 2]]. On the TI-82/83 the sum of the two vectors will appear as follows:

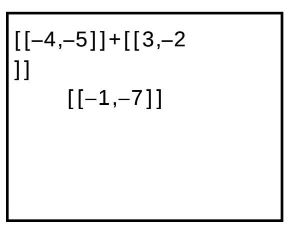

Figure 14

We see that < – 4, – 5 > + < 3, – 2 > = < – 1, – 7 >.

Exercises:

Use your calculator to find the sums of the following vectors.

10) < 2, 3 > + < 3, 5 >

11) < 3, 4 > + < – 2, 2 >

12) < 0, 4 > + < – 4, – 1 >

13) < – 1, – 1 > + < – 3, 2 >

Subtraction of Vectors

Subtraction of wind velocity vectors is used to determine if the wind has changed too much since the guidance system was programmed so that the launch should be aborted. In the same way we subtract two integers by adding the additive inverse of the second integer, we subtract two vectors by changing it to an addition problem. Recall that the additive inverse of an integer, a, is a number, – a, such that a + (– a) = 0. In the same way the additive inverse of the vector < 2, – 3 > is a vector which when added to < 2, – 3 > yields the zero vector < 0, 0 >. Figure 15 shows the vector **A** = < 2, – 3 > and its additive inverse. Note that if we add these two vectors the resultant vector is the zero vector < 0, 0 >.

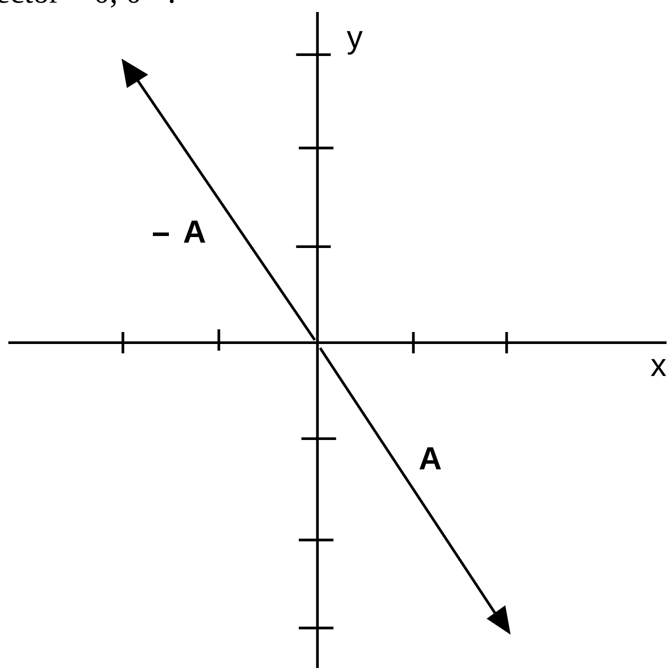

Figure 15

Question:

8) Look at Figure 15. Write the additive inverse of < 2, – 3 > in < x, y > form.
How can you find the additive inverse of < 2, – 3 > without graphing the vector?

We can see that the additive inverse of < 2, – 3 >, the vector – < 2, – 3 >, is equal to < – 2, 3 >. This is because < 2, – 3 > + < – 2, 3 > = < 0, 0 >, the zero vector. (In fact, if we multiply a vector < x, y > by ANY scalar k we have k< x, y > = < kx, ky >.)

Figure 16 shows how we can find < 4, 5 > – < 2, – 3 > by finding the sum
S =< 4, 5 > + < – 2, 3 >. Write the sum in < *x*, *y* > form.

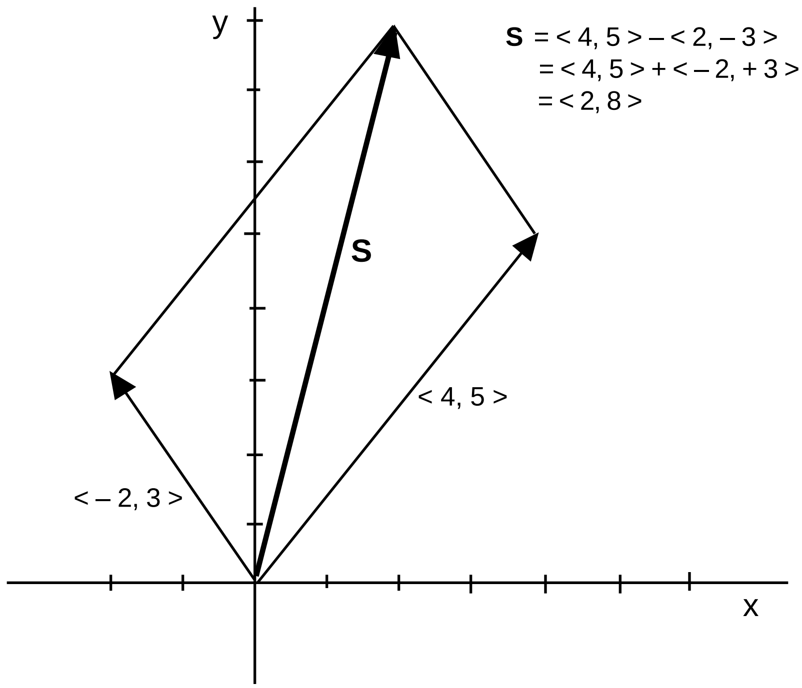

Figure 16

We can use graphing calculators to subtract two vectors in the same manner that we add two vectors. Simply replace the addition key with the subtraction key when entering the problem. For example, < 4, 5 > – < 2, – 3 > = < 2, 8 > appears on the TI-82™/83™ as follows:

```
[[4,5]]-[[2,–3]]
        [[2,8]]
```

Figure 17

Exercises:

Use your graphing calculator to find each difference.

14) < 2, 3 > – < 3, 5 >

15) < 3, 4 > – <– 2, 2 >

16) < 0, 4 > – < – 4, – 1 >

17) < – 1, – 1 > – <– 3, 2 >

Look at the resultant vectors. Do you see a pattern? Describe in your own words how to subtract two vectors in rectangular coordinates without graphing them.

Building The Model

(Back to the DRWP)

The Doppler Radar Wind Profiler provides the meteorologist with two components, u and v, of the wind velocity. The letter u represents the wind component in the east-west direction with east taken as the positive direction, and v represents the wind component in the north-south direction with north being positive. A coordinate system is used to establish a frame of reference. It is common to orient the coordinate system so that the positive x-axis points in the east direction and the positive y-axis points in the north direction. If the east component is 5 m/s and the north component is – 3 m/s (the negative means the wind direction is south), the wind velocity can be represented by the vector $< 5, -3 >$.

Exercises:

18) Find a rectangular vector to represent the wind velocity **A** for each pair of components given in Table 1 below. These component readings come from gate 30 at $t-4$, $t-2$, $t-1$, and $t-0.25$ hours, respectively.

19) Use a calculator or computer program to change each vector in column 4 of Table 1 to a polar vector. The magnitude r of this vector is the speed of the wind. This number is also called the **norm** of the vector. The angle θ tells us the direction of the wind.

Table 1

Time	u	v	Wind velocity vector **A** (rectangular form)	Wind velocity vector **A** (polar form)
t - 4	13 m/s	5 m/s		
t - 2	-3 m/s	6 m/s		
t - 1	-5 m/s	8 m/s		
t - 0.25	-7 m/s	4 m/s		

In order to decide if the launch should be aborted or not, the operator must look at wind change in the plane of launch and wind change perpendicular to this plane. These components of the velocity vector are called the **in-plane** and **out-of-plane** components. The in-plane is the vertical plane in which the Shuttle trajectory lies. When the shuttle is launched, the direction in which it is launched depends on the mission. Common launch directions are directly east (0˚ north of east), 35˚ north of east, and 56˚ north of east. If the launch direction is 0˚, the east component u is the in-plane component and the north component v is the out-of-plane component. This is the easiest case, since we do not have to rotate the axes to find the desired components. (See Figure 18)

Gates and a Due East Shuttle Trajectory

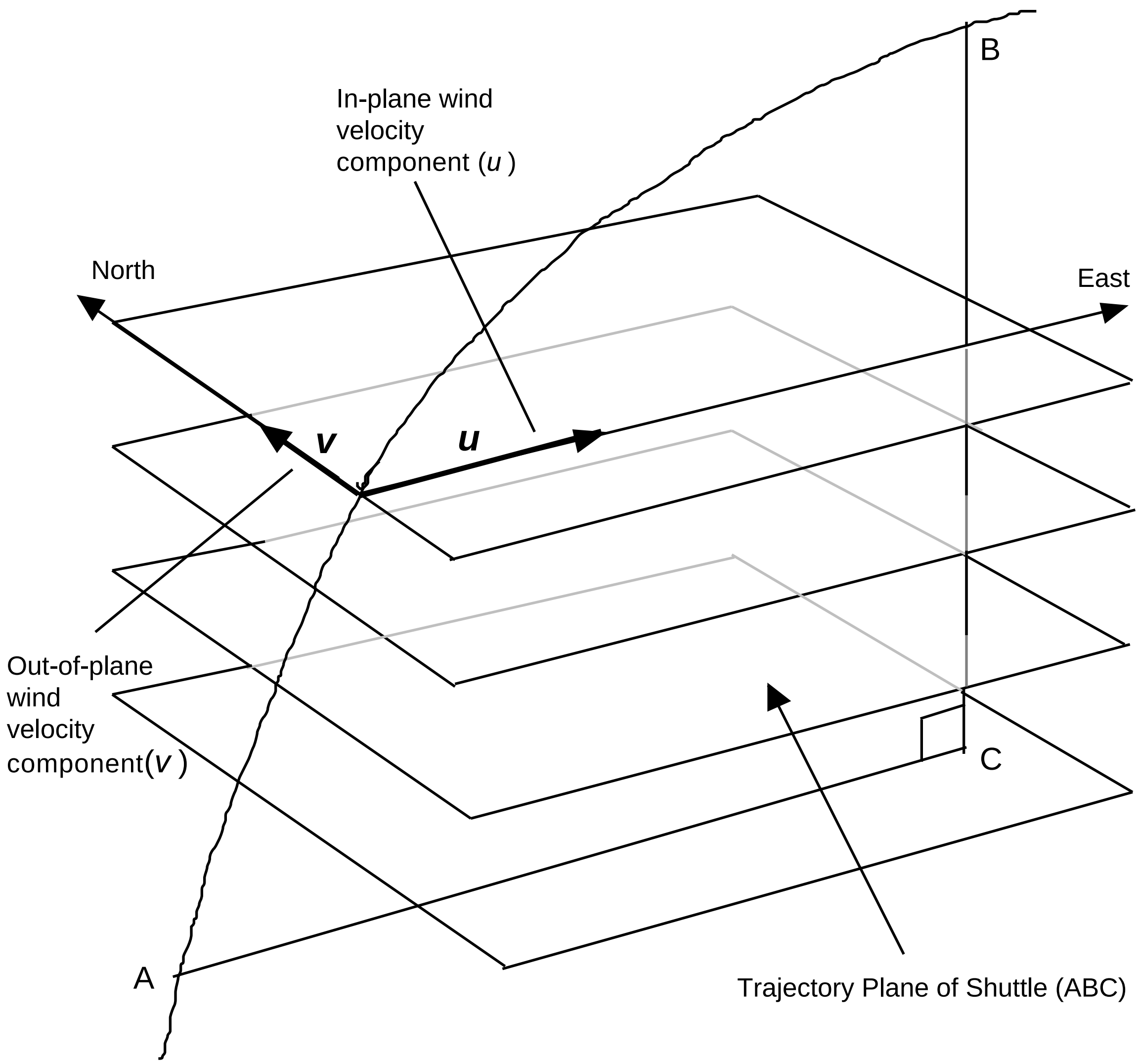

Figure 18

Example 7**:** Suppose the DRWP readings at gate 40 at time $t - 4$ hours were $u = 4$ m/s and $v = -2$ m/s. Therefore, the horizontal velocity vector $\mathbf{A} = \langle u, v \rangle$ at $t - 4$ hours is $\langle 4, -2 \rangle$. This vector along with the readings at the other 111 gates was used to create a guidance program. The reason for using these data is that it takes almost 4 hours to run the program to determine the loads, create a guidance program to insure a safe launch under these conditions, and load the program. If the readings at gate 40 at time $t - 1$ hours were $u = 20$ m/s and $v = 16$ m/s, is this wind change drastic enough to delay or scrub the launch?

Solution**:**
The wind velocity vector **A** at time $t - 1$ hours was $\langle 20, 16 \rangle$, and at $t - 4$ hours the value of **A** was $\langle 4, -2 \rangle$. The change in wind velocity is the difference between these values of **A**.

$$\Delta\mathbf{A} = \langle 20, 16 \rangle - \langle 4, -2 \rangle = \langle 16, 18 \rangle$$

The threshold value for changes in wind speed in-plane and out-of-plane is approximately 20 m/s. Looking at the vector for the change in wind velocity we see that the absolute values of the two components are 16 in-plane and 18 out-of-plane. We are still within the safety envelope and do not need to delay the launch to recalculate the guidance program (if the launch window is over 3 hours) or scrub the launch (if the launch window is less than 3 hours).

Question:

9) Suppose the readings at gate 62 at $t - 4$ hours were $u = -5$ m/s and $v = 13$ m/s while at $t - 1$ hour the readings at gate 62 were $u = -5$ m/s and $v = -8$ m/s. Should the launch be delayed or scrubbed based on these readings? Justify your answer.

Mathematical Aside: Rotation of Axes

If the launch path is not directly east as shown in Figure 18, we must find the components of the wind velocity in the new direction and perpendicular to the new direction. Figure 19 shows a launch path 35˚ north of east.

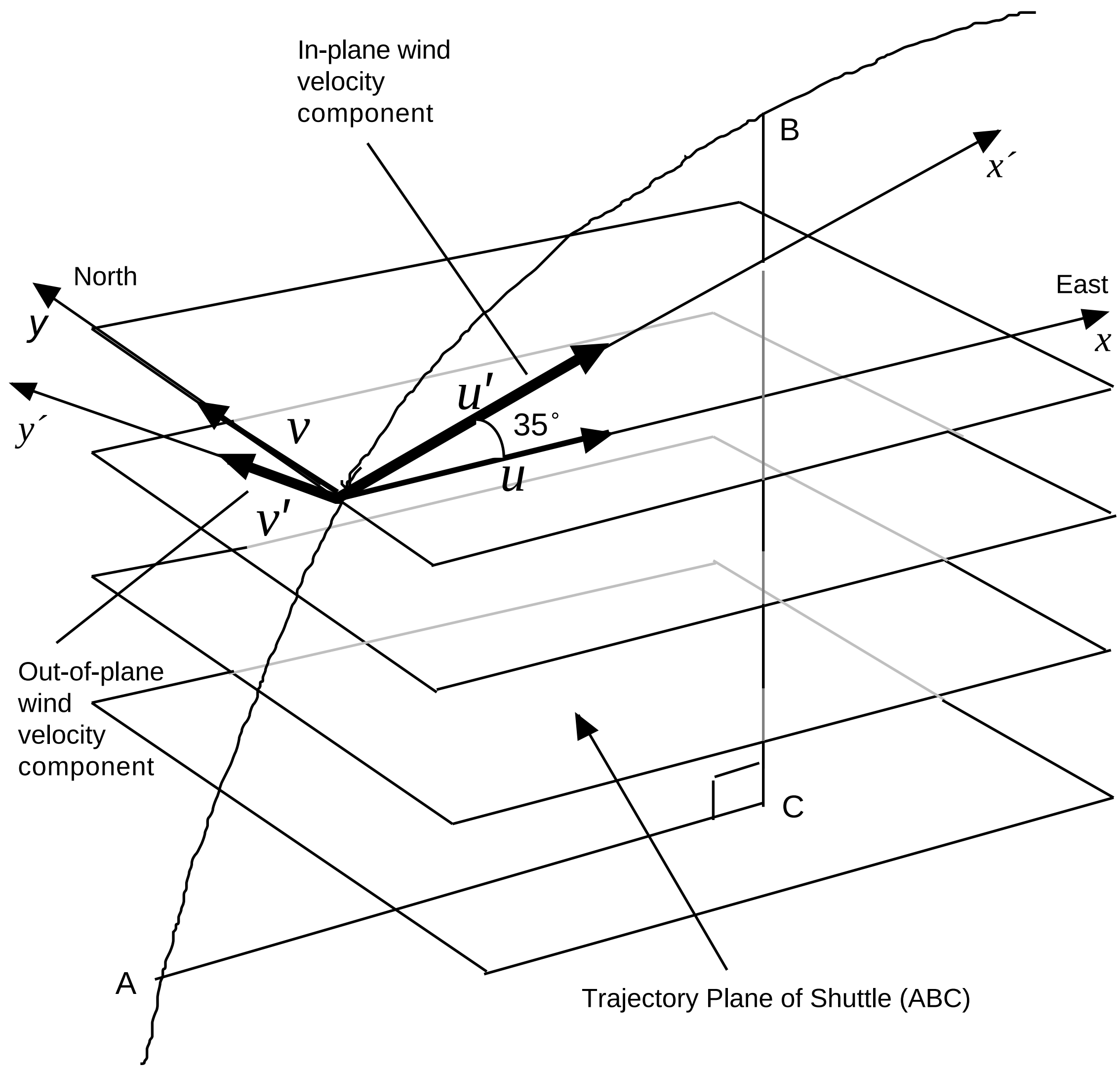

Figure 19

The easiest way to visualize the new components (u' for in-plane and v' for out-of-plane) is as a rotation of the x and y axes for each gate through the appropriate angle. We then consider the components of the wind velocity vector along these new axes. This rotation is shown in Figure 20. The new coordinate system has been rotated φ degrees from the original axes, where φ is the angle of the direction of launch north of directly east.

Rotation of Axes

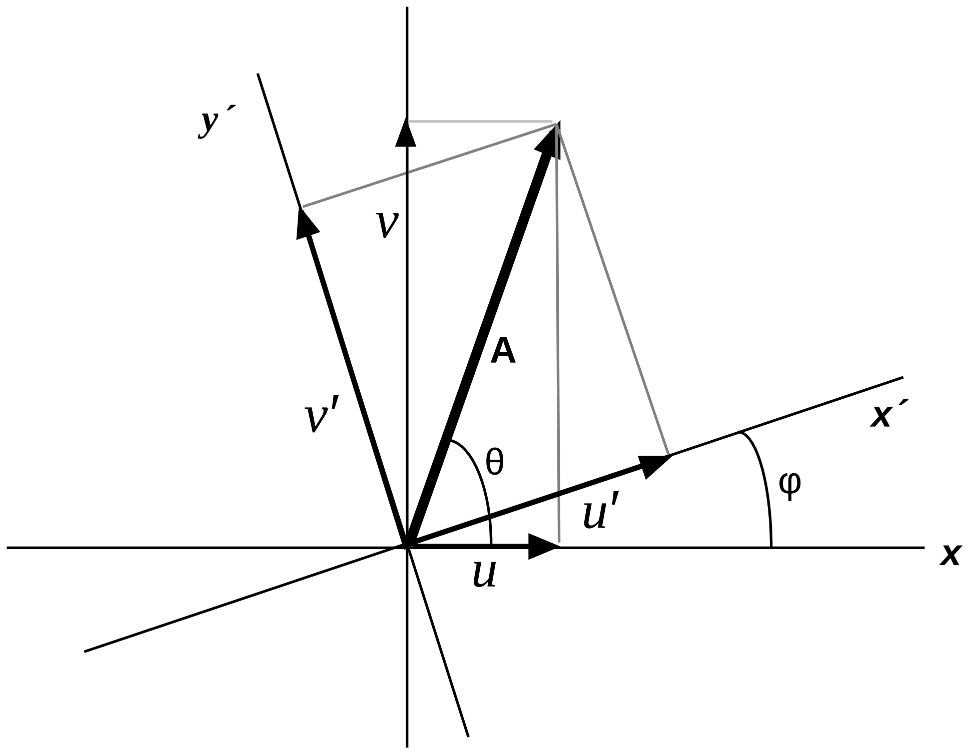

Figure 20

Example 8: Estimate the u' and v' components for the given vector **A** = < 2, 6 > and the given rotation in Figure 20.

Solution: The component u' in the direction of the x' axis appears to be two times the length of u or $u' = 4$. The component v' in the direction of the y' axis appears to be one less than the length of v or $v' = 5$. The vector < 4, 5 > is a reasonable estimate for the vector **A** in the rotated coordinate system.

From Figure 20 we can see that if r is the length of the vector **A**, then

$$u' = r\cos(\theta - \varphi) = r\left[\cos(\theta)\cos(\varphi) + \sin(\theta)\sin(\varphi)\right]$$
$$= r\cos(\theta)\cos(\varphi) + r\sin(\theta)\sin(\varphi)$$

But $r\cos(\theta) = u$ and $r\sin(\theta) = v$. Substituting, we get a formula for finding the new component in the direction of the x' axis. We use similar reasoning to obtain a formula for finding the v' component of the vector **A**. The components of a vector in the new coordinate system can be found by using the following formulas:

$$u' = u\cos(\varphi) + v\sin(\varphi) \qquad\qquad v' = -u\sin(\varphi) + v\cos(\varphi)$$

The following TI-83™ program will compute the components for the rotated axes for you. Note: Do not confuse the letter "O" used in the program to represent the out-of-plane component with the number zero.

Program 3: Conversion from xy coordinates to $x'y'$ coordinates

```
PROGRAM:VECT3
:ClrHome
:Degree
:Fix 2
:Disp "ENTER U"
:Prompt U
:Disp "ENTER V"
:Prompt V
:Disp "ENTER PHI"
:Prompt P
:U*cos(P)+Vsin(P)→I
:–U*sin(P)+V*cos(P)→O
:ClrHome
:Disp "IN–PLANE:"
:Disp I
:Disp "OUT–OF–PLANE:"
:Disp O
```

Example 9: Find the components of the vector < 3, 2 > in a coordinate system rotated 40˚ from the original set of axes.

Solution:

$$u' = 3\cos(40^\circ) + 2\sin(40^\circ) = 3.58$$
$$v' = -3\sin(40^\circ) + 2\cos(40^\circ) = -0.40$$

Thus, the vector in the new coordinate system would be < 3.58, – 0.40 >

On the TI-83™, Program 3 produces the following screens:

```
ENTER U
U=?3
ENTER V
V=?2
ENTER PHI
P=?40
```

Input: U, V, φ

```
IN–PLANE
        3.58
OUT–OF–PLANE
        –.40
```

Output: In-Plane, Out-of-Plane

Solving the Case Where the Launch Path is not Directly East

Questions:

10) Table 2 gives the DRWP readings at $t - 1$ hour for several different gates. As usual u and v represent the components of the wind velocity in the east and north directions, respectively. Assume that the launch path will be at $\varphi = 35^\circ$. For each set of readings, compute the components, u' and v', of the wind velocity for the rotated axes.

Table 2

u	v	u'	v'
11 m/s	2 m/s		
5 m/s	-3 m/s		
1 m/s	-6 m/s		
-3 m/s	-2 m/s		

11) Table 3 shows the DRWP readings at the same gates at $t - 4$ hours. Compute the wind velocity components for the rotated axes for these initial readings.

Table 3

u	v	u'	v'
4 m/s	7 m/s		
-6 m/s	12 m/s		
16 m/s	9 m/s		
-21 m/s	5 m/s		

Exercises:

20) Find the change in velocity at each gate (using the rotated components) by finding the difference in the corresponding vectors. Should the launch proceed or should it be delayed or scrubbed? Justify your answer.

21) Repeat this investigation, but change the launch path to 56°.

KSC's Doppler Radar Wind Profiler (DRWP)

Exercise:

22) Assume that the the two oblique beams of the DRWP at the Kennedy Space Center are not directed east and north, but are rotated – 45˚. This means that the two wind velocity components found at each gate are not east and north components. Instead the components are measured relative to a coordinate system with axes that we will label *a* and *b*. Moreover, if the *a*-*b* system is rotated 45˚ counterclockwise the *a* and *b* axes will coincide with the *u* (east) and *v* (north) respectively. Therefore, the data must be converted to *u* (east) and *v* (north) components, and then the approach described previously is used to determine whether or not to launch. This conversion involves a rotation of 45˚. Use what you have just learned about rotations of axes to convert the KSC components *a* and *b* for time *t* – 1 hour to the standard components *u* and *v*.

Table 4

a	*b*	*u*	*v*
4 m/s	7 m/s		
-6 m/s	12 m/s		
16 m/s	9 m/s		
-21 m/s	5 m/s		

Suppose the wind velocity vectors in < *u*, *v* > format for the same gates at time *t* – 4 hours were < 1, – 12 >, < 0, 20 >, < 7, 2 >, and < 2, 3 >. Should a shuttle flight directly east be delayed or scrubbed based on the change in wind velocity over this time period? Justify your answer.