

# ***FACULTY NOTES***

The LTAs and Spinoffs are designed so that each professor can implement them in a way that is consistent with his/her teaching style and course objectives. This may range from using the materials as out-of-class projects with minimal in-class guidance to doing most of the work in class. The LTAs and Spinoffs are amenable to small group cooperative work and typically benefit from the use of some learning technology. Since the objective of the LTAs and Spinoffs is to support the specific academic goals you have set for your students, the Faculty Notes are not intended to be prescriptive. The purpose of the Faculty Notes is to provide information that assists you to take full advantage of the LTAs and Spinoffs. This includes suggestions for instruction as well as answers for the exercises.



## FACULTY NOTES

### LTA 3

#### **The Doppler Radar Wind Profiler: Vector Analysis of Wind Changes Affecting Shuttle Launch at the Kennedy Space Center**

#### **Background Information**

##### Learning Objectives:

The student will be able to:

- perform basic vector algebra including addition, subtraction, and scalar multiplication
- convert between rectangular and polar coordinate systems
- use vectors to solve problems relating to changes in wind velocity
- rotate axes using appropriate trigonometric formulas

##### Mathematical Prerequisites:

Algebra  
Cartesian and polar coordinates  
Knowledge of trigonometric functions  
Introduction to vectors useful but not essential

##### Mathematics Topics:

Cartesian coordinates  
Polar coordinates  
Trigonometry  
Rotation of axes  
Vector Algebra

##### Technical Applications:

Description of the Doppler Radar Wind Profiler (DRWP)  
Analysis of wind factors affecting Shuttle safety  
Factors affecting launch angle (orbit, physical conditions at site)

##### Learning Technologies Suggested:

Programmable scientific calculator, graphing calculator  
Material is well suited for implementation using a computer algebra system (CAS).

##### Time Frame for Implementation:

- One or two 1.5 hour class periods depending on level of students
- With no vector algebra background: 1 period on introduction to vectors + 1 period on DRWP application
  - With vector background: 1 period on review of vectors + DRWP application

Note: Preprogramming of graphing calculator will reduce time.

##### Instructional Methodologies:

Short lecture  
Small groups  
Exploring ideas with the graphing calculator

### Assessment:

Assessment exercises are integrated into the LTA. Students are assessed on the basis of:

- numerical solutions to routine computational exercises,
- ability to reason mathematically,
- ability to communicate results,
- ability to make decisions regarding the Shuttle that are based on quantitative reasoning.

Suggested Course(s): College Algebra, Precalculus

Math Topics(s): Vectors, trigonometry

## Solutions

### Questions

- 1) Gusts, squalls, storms, tornadoes, hurricanes
- 2) Sudden changes in wind speed (gusts)  
Sudden changes in wind direction (eddies, swirls, vortexes)
- 3)  $t - 4$  profile is taken at 6:00 a.m.  
 $t - 1$  profile is taken at 9:00 a.m.  
New launch time is 1:00 p.m.
- 4) Scrub the launch because the launch window is only 1.5 hours, and it takes 4 hours to recalculate the guidance program.

Note: The angle as defined in this document is consistent with common mathematical usage. However, professionals who work with weather (wind) data use a different convention as described in the following quote from NASA scientist Francis Merceret.

“Actually, for weather (wind) data, the direction of is the direction from which the wind is blowing referenced to the north. If  $u$  is the component toward the east, and  $v$  is the component toward the north, then

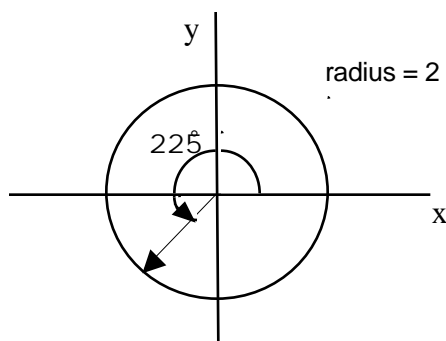
$$u = -ws \sin(wd)$$

$$v = -ws \cos(wd)$$

where  $ws$  is the wind speed and  $wd$  is the wind direction”

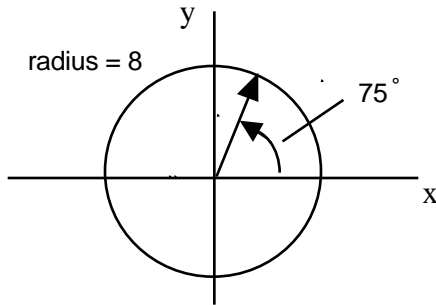
5)  $\langle 3, 70^\circ \rangle$

6)

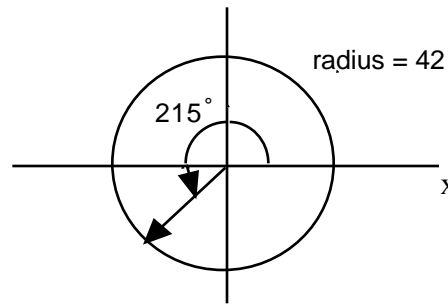


## Exercises

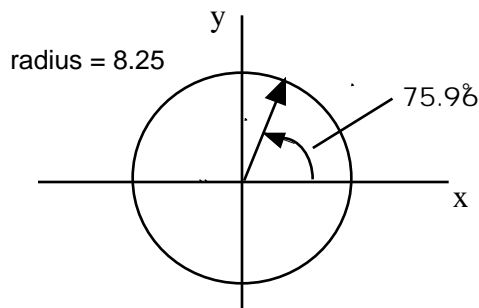
1)  $x = 8 \cos(75^\circ) = 2.07$   
 $y = 8 \sin(75^\circ) = 7.73$



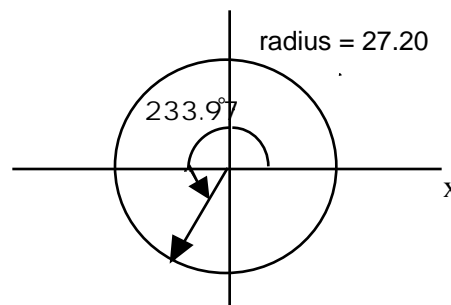
2)  $x = 42 \cos(215^\circ) = -34.40$   
 $y = 42 \sin(215^\circ) = -24.09$



3)  $r = \sqrt{4 + 64} = 8.25$   
 $= \tan^{-1} \frac{8}{2} = 75.96^\circ$



4)  $r = \sqrt{256 + 484} = 27.20$   
 $= \tan^{-1} \frac{22}{16} + 180^\circ = 233.97^\circ$



5)  $x = 3 \cos(120^\circ) = -1.50$   
 $y = 3 \sin(120^\circ) = 2.60$

6) Select any vector with  $270^\circ < \theta < 360^\circ$ . For example, the vector  $\langle 10, 300^\circ \rangle$  in polar form becomes  $\langle 5.00, -8.66 \rangle$  in rectangular form.

7)  $r = \sqrt{64 + 36} = 10$   
 $= \tan^{-1} \frac{6}{-8} + 180^\circ = -36.87^\circ + 180^\circ = 143.13^\circ$

8) Because  $x < 0$  and  $y > 0$  so that the angle is in quadrant 2.

9) Choose any vector with  $x > 0$  and  $y < 0$  (i.e. in quadrant 4). For example, the vector  $\langle 10, -8 \rangle$  in rectangular form becomes  $\langle 12.81, 321.34^\circ \rangle$  in polar form.

### Question

7)  $\langle 7, -1 \rangle$   
 $\langle 3, 6 \rangle$

Add the  $u$  components together to get the summed  $u$  value; similarly add the  $v$  components together to get the summed  $v$  value.

### Exercises

10)  $\langle 5, 8 \rangle$

11)  $\langle 1, 6 \rangle$

12)  $\langle -4, 3 \rangle$

13)  $\langle -4, 1 \rangle$

### Question

8) Additive inverse of  $\langle 2, -3 \rangle$  is  $\langle -2, 3 \rangle$

Obtain additive inverse by changing the signs of both  $x$  and  $y$ .

### Exercises

14)  $\langle -1, -2 \rangle$

15)  $\langle 5, 2 \rangle$

16)  $\langle 4, 5 \rangle$

17)  $\langle 2, -3 \rangle$

Subtract the first components to get the  $u$  component of the difference; similarly subtract the second coordinates to get the  $v$  component of the difference.

18)  $\langle 13, 5 \rangle$   
 $\langle -3, 6 \rangle$   
 $\langle -5, 8 \rangle$   
 $\langle -7, 4 \rangle$

19)  $\langle 13.93, 21.04^\circ \rangle$   
 $\langle 6.71, 116.57^\circ \rangle$   
 $\langle 9.43, 122.01^\circ \rangle$   
 $\langle 8.06, 150.26^\circ \rangle$

### Questions

9)  $= \langle -5, -8 \rangle - \langle -5, 13 \rangle = \langle 0, -21 \rangle$

Since the absolute value of the out-of-plane wind change is 21 which exceeds the threshold value of 20, the launch should be delayed if the launch window exceeds 3 hours, or scrubbed if the launch window is less than 3 hours.

10)

**Table 2**

$u'$	$v'$
10.16	- 4.67
2.38	- 5.33
- 2.62	- 5.49
- 3.60	0.08

11)

**Table 3**

$u'$	$v'$
7.29	3.44
1.97	13.27
18.27	-1.80
- 14.33	16.14

**Exercises**

20) At the four gates the vector changes are:

- < 2.87, - 8.11 >
- < 0.41, - 18.60 >
- < - 20.89, - 3.69 >
- < 10.73, - 16.06 >

The in-plane value at the third of these gates is - 20.89. Since its absolute value exceeds the threshold of 20 the launch should be delayed or scrubbed.

21) **<  $u'$ ,  $v'$  > at  $t - 1$  hour**      **<  $u'$ ,  $v'$  > at  $t - 4$  hours**      **The differences are:**

- < 7.81, - 8.00 >                      < 8.04, 0.60 >                      < - 0.23, - 8.60 >
- < 0.31, - 5.82 >                      < 6.59, 11.68 >                      < - 6.28, - 17.5 >
- < - 4.42, - 4.18 >                      < 16.41, - 8.23 >                      < - 20.83, 4.05 >
- < - 3.34, 1.37 >                      < - 7.60, 20.21 >                      < 4.26, - 18.84 >

The third one of the gates has an in-plane change that exceeds 20. Therefore, delay or scrub.

22) It is important to note that the shuttle flight path is due east. This means that the in-plane wind velocity ( $u'$ ) and the out-of-plane wind velocity ( $v'$ ) are equal to the east ( $u$ ) and north ( $v$ ) components of the wind velocity respectively.

**At  $t - 1$  we get:**                      **At  $t - 4$  we are given:**                      **The differences are:**

- < 7.78, 2.12 >                      < 1, - 12 >                      < 6.78, 14.12 >
- < 4.24, 12.73 >                      < 0, 20 >                      < 4.24, - 7.27 >
- < 17.68, - 4.95 >                      < 7, 2 >                      < 10.68, - 6.95 >
- < - 11.31, 18.38 >                      < 2, 3 >                      < - 13.31, 15.38 >

Since all differences are less than the threshold value of 20, there is no need to delay or scrub.