SPINOFFS

Spinoffs are relatively short learning modules inspired by the LTAs. They can be easily implemented to support student learning in courses ranging from prealgebra through calculus. The Spinoffs typically give students an opportunity to use mathematics in a real world context.

LTA - SPINOFF 2A	Cost Analysis for the NASA Aquatics Lab
LTA - SPINOFF 2B	NASA Aquatics Lab Loading

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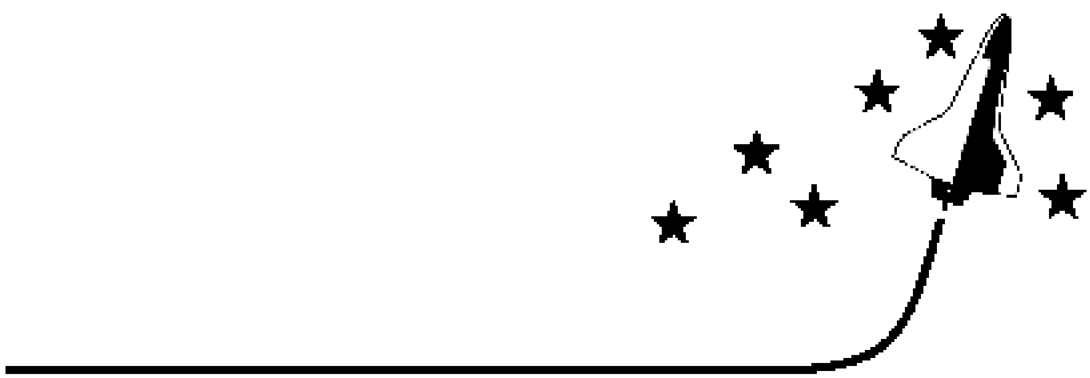

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SPINOFF 2B

Aquatics Lab Loading

Background Information

French and German scientists used space aboard the Shuttle Columbia for an August 1997 launch. They had constructed experiments to study the effects of weightlessness on fish and snails.

An aquatics lab was built at Kennedy Space Center which was used to perform preliminary experiments in preparation for the launch. Unused office space on the second floor of an existing building was modified as shown on the floor plan on the last page of this Spinoff. The plan shows a number of racks of aquaria of different sizes which were distributed as shown in the plan.

Student Task

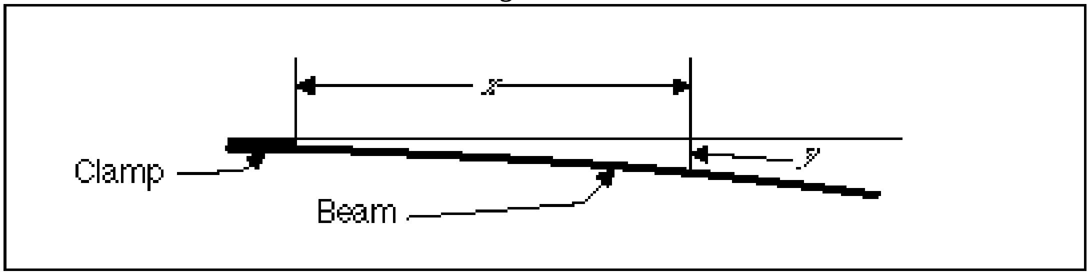
You are to calculate the maximum deflection of a beam under stress due to the weight of racks of 50 gallon aquaria.

Technical Information

The relation between weight on a horizontal beam and the amount of bending which results can be understood by considering a simple example. Suppose a person is standing on a diving board above a pool. A diving board is a horizontal beam supported at one end. The further out a person stands from the supported end of the diving board, the greater the bending of the board. Experiments show that, within the elastic limits of the material, this relation is linear. That is, the effect of the weight, w, on the bending of a diving board is proportional to wx, where x is the distance measured from the supported end of the diving board. The product wx is called a moment of force, often measured in ft-lbs.

Let x be the horizontal distance along a beam, and let y be the vertical deflection of the beam (Figure 1).

Figure 1



The amount of bending of the beam is given by the formula for curvature, κ.

$$\kappa = \frac{\dfrac{d^2y}{dx^2}}{\left(1+\left(\dfrac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}$$

When κ is small there is very little bending.

In actual building construction $\left|\frac{dy}{dx}\right|$ is assumed to be very small compared to 1 so that $1+\left(\frac{dy}{dx}\right)^2 \approx 1$, and we then have $\kappa \approx \frac{d^2y}{dx^2}$. Since the amount of bending is directly proportional to the moment about x, $M(x)$, we have $\frac{d^2y}{dx^2} \quad M(x)$. To get an equation, a constant of proportionality is needed. In this case, the constant depends on the physical characteristics of the material that the beam is made of and its cross-sectional area. Without going into the details, it turns out that the formula used is:

$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI} \qquad \text{(Equation 1)}$$

where E is the modulus of elasticity of the material used and I is the moment of inertia about an axis through the center of mass of a cross-section of the beam.

Sample Calculation: *M(x)* and *y(x)*

Suppose a 150 pound weight is located 6 ft from end A and 3 ft from end B on a wooden beam as shown below.

Suppose further that both ends of the beam are supported. Since this is a static situation, the sum of the moments about any point must be zero. From statics, the reactive forces at A and B, R_A and R_B respectively, must balance the 150 lb weight.

Choosing A as one point, the moment equation is:

R_B(9 ft) – (150 lb)(6 ft) = 0. (That is, the sum of the moments about A is zero.)

Then, R_B = 100 lb.

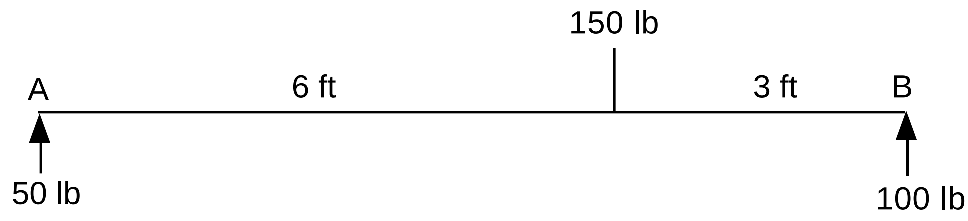
Choosing B as another point, the moment equation is, similarly:

(150 lb)(3 ft) – R_A(9 ft) = 0.

Then, R_A = 50 lb.

(In these calculations, a counter-clockwise moment was arbitrarily chosen as positive and a clockwise moment was negative. Also, we will consider the upward direction positive and the downward direction negative. The reactive forces act in opposition to the 150 lb weight.)

So, with the reactive forces calculated, the diagram of our sample situation is shown below:



Now, to compute the moment, $M(x)$, about a variable point x, we let $x = 0$ at A. Then,

for $0 \le x \le 6$, $M(x) = 50x$, and
for $6 \le x \le 9$, $M(x) = 50x - 150(x-6)$.

Note: $M(0) = M(9) = 0$; i.e., the bending is zero at the ends where the beam is supported.

Using Equation 1, integrating *M(x)* twice and using boundary conditions to evaluate the constants of integration would give the deflection y in terms of x. However, since *M(x)* is piecewise defined, if each piece were to be integrated separately, there would be four constants of integration to evaluate. To alleviate this cumbersome procedure, *M(x)* can be written as $M(x) = 50x - 150 < x - 6 >$, where $< x - 6 >$ is itself piecewise:

$$< x - 6 > = \begin{cases} x - 6 & \text{if } x \ge 6 \\ 0 & \text{if } x < 6 \end{cases}$$

In general, $< x - a >^n = \begin{cases} (x-a)^n & \text{if } x \ge a \\ 0 & \text{if } x < a \end{cases}$ are called **singularity** functions. Using a singularity function, only two constants of integration need to be calculated. Thus,

$$M(x) = EI\frac{d^2y}{dx^2} = 50x - 150 < x - 6 >,$$

$$EI\frac{dy}{dx} = 25x^2 - 75 < x - 6 >^2 + C_1, \text{ and}$$

$$EI\,y(x) = \frac{25}{3}x^3 - 25 < x - 6 >^3 + C_1 x + C_2$$

Since $y(0) = y(9) = 0$ (the beam is supported at both ends), we get $C_1 = -600$, and $C_2 = 0$. Therefore,

$$EI\,y(x) = \frac{25}{3}x^3 - 25 < x - 6 >^3 - 600x, \quad 0 \le x \le 9 \qquad \text{(Equation 2)}$$

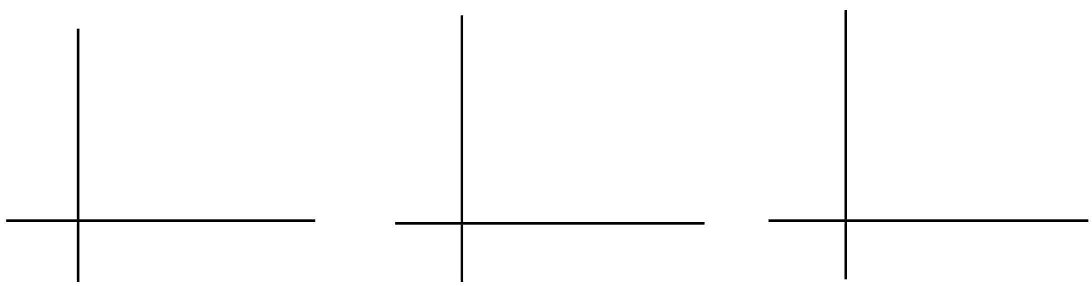
Recall, $< x - 6 >$ is zero when $x < 6$. Using this fact and the first derivative test, it can be shown that the maximum deflection occurs at $x = 4.90$ ft. Consider, for example, a wooden beam with cross sectional dimension 2" x 4" resting on its edge. For this situation, a table of values for the modulus of elasticity gives E 1.6×10^6 lb/in^2, and a moment of inertia, I 6.25 in^4. At $x = 4.90$ ft, the amount of the maximum deflection is, from Equation 2, $y = 0.339$ inches.

Note: Since the units of $M(x)$ are ft-lbs, the right-hand side of Equation 2 has dimensions lb-ft^3, so you must convert the result of the calculation on the right-hand side to lb-in^3 in order to obtain the correct answer.

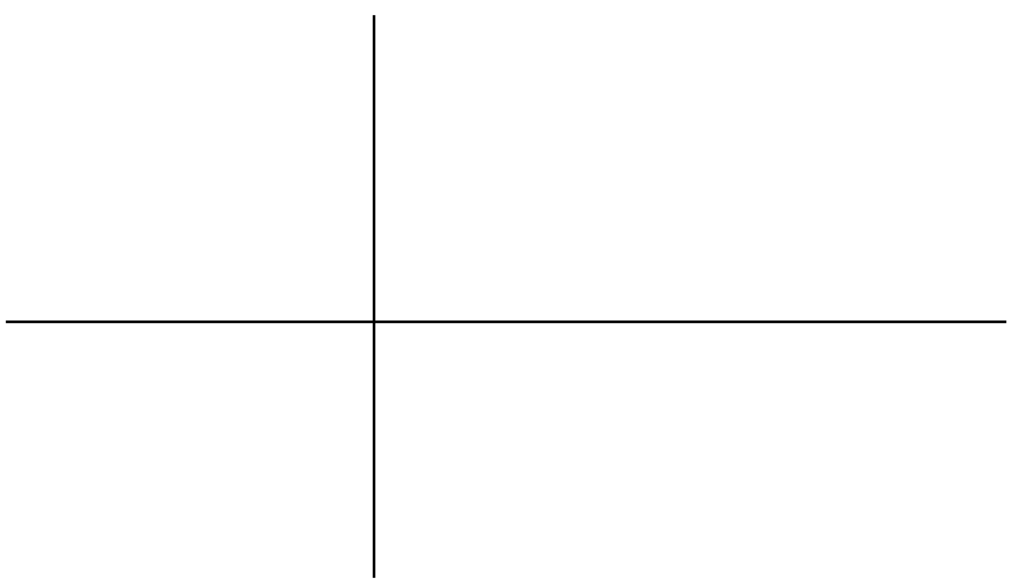
Worksheet

Part I: Singularity Functions

1) On the coordinate axes provided below, sketch the graphs of $< x - a >^0$, $< x - a >^1$, and $< x - a >^2$, for $a > 0$.



2) Using Equation 2, sketch the graph of $y(x)$ on [–10, 15]. Use feet for the units on the x-axis and inches for the units on the y-axis.



3) In the derivation of a formula for the deflection, *y(x)*, the approximation $1+\left(\frac{dy}{dx}\right)^2 \approx 1$ was made. Use your calculator/software to verify that this approximation is reasonable by graphing $\frac{dy}{dx}$ on [0,9]. Then, calculate $1+\left(\frac{dy}{dx}\right)^2$ for the largest possible value and compare it to 1.

largest value of $1+\left(\frac{dy}{dx}\right)^2$: ______

4) The function *y(x)* in Equation 2 contains a singularity function. Is *y(x)* differentiable on the interval (0, 9)? Explain your answer.

Part II: Unit Analysis

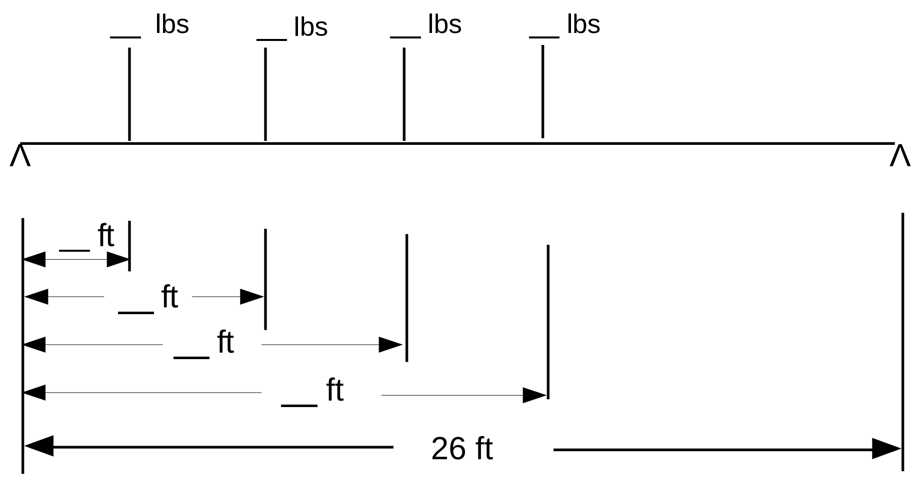
1) In the sample calculation it was stated that the unit of the terms on the right-hand side of Equation 2 was lb-ft^3. Explain why the unit is lb-ft^3. Hint: *M(x)* was integrated twice to obtain *y(x)*.

2) In the sample calculation, the numerical value of C_1 turned out to be – 600. What are the units that need to be assigned to C_1 in order for Equation 2 to make sense? Explain your reasoning.

Part III: Calculating *y(x)*

Referring to the floor plan on the last page of this Spinoff, the third row of aquaria from the right-hand side consists of four racks of 50 gallon aquaria. The aquaria are stacked three high in a rack. Water weighs 8.34 lb/gal, and each rack which holds three aquaria weighs 250 lb. As a convenient approximation, suppose that the combined weight of water and rack is concentrated at the geometric center of the rectangles shown in the floor plan.

1) Assume the row of 50 gallon aquaria rests on a reinforced concrete beam 26 feet long. Use a ruler to estimate to the nearest 1/16 inch (i.e., the nearest 1/2 foot), the distance from the wall at the top of the page to the geometric center of each rectangular rack. (Use the outside of the double line in the floor plan as the end of the beam.) Fill in the distances and weights on the static diagram shown below. Also, calculate the reactive forces at the beam ends as outlined in the sample calculation, and fill in the reactive force values at the beam ends in following diagram.



2) Using the data from the diagram, calculate *y(x)* for this situation. You will have four singularity functions in your answer. Also, state the boundary conditions which are needed to determine the constants of integration.

Boundary conditions: $y(0) = y(26) =$ ______

$EIy(x) =$ ____

Note: Before proceeding, verify your solution above with at least one other group member or with your instructor.

3) For the reinforced concrete beam, use $E = 6 \times 10^6$ lb/in^2 and $I = 700$ in^4. Differentiate *y(x)*, and use the result to calculate the location and the amount of the maximum deflection of the beam.

x-value (location): __________

Maximum deflection: __________

Note: A rule of thumb for structural engineering is that the maximum deflection should not be more than the length of the beam in inches divided by 360. With respect to this rule of thumb, does your answer seem to be "in the ballpark" ?

Part IV: A Safety Problem

When NASA designers inspected the lab as equipment was being installed, there was an extra water tank, cylindrical in shape, 4 feet in diameter and 6 feet high, located near the outside wall at point A on the floor plan. The purpose of the extra tank was to cool outside water and provide a source of "settled" water for the aquaria and related experiments. Unfortunately, the tank capacity was far too large for the reinforced concrete floor on which the tank was standing. From original blueprints and building specifications, a NASA engineer calculated that the floor could safely support no more than 200 lb/sq ft.

Calculate the distributed load of the full tank resting on the floor with no special support, and write a concluding statement which indicates a reasonable and simple solution to the problem caused by the presence of the extra tank.

REFERENCE:

Beer and Johnston, Jr.: Mechanics of Materials, 2nd ed., Section 7.5, pp. 432-434, McGraw-Hill, Inc. 1992.