

FACULTY NOTES

The LTAs and Spinoffs are designed so that each professor can implement them in a way that is consistent with his/her teaching style and course objectives. This may range from using the materials as out-of-class projects with minimal in-class guidance to doing most of the work in class. The LTAs and Spinoffs are amenable to small group cooperative work and typically benefit from the use of some learning technology. Since the objective of the LTAs and Spinoffs is to support the specific academic goals you have set for your students, the Faculty Notes are not intended to be prescriptive. The purpose of the Faculty Notes is to provide information that assists you to take full advantage of the LTAs and Spinoffs. This includes suggestions for instruction as well as answers for the exercises.



FACULTY NOTES

SPINOFF 2B

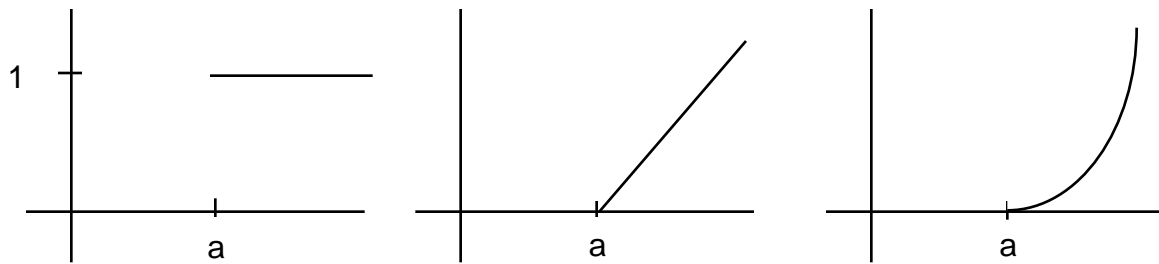
Aquatics Lab Loading

Solutions

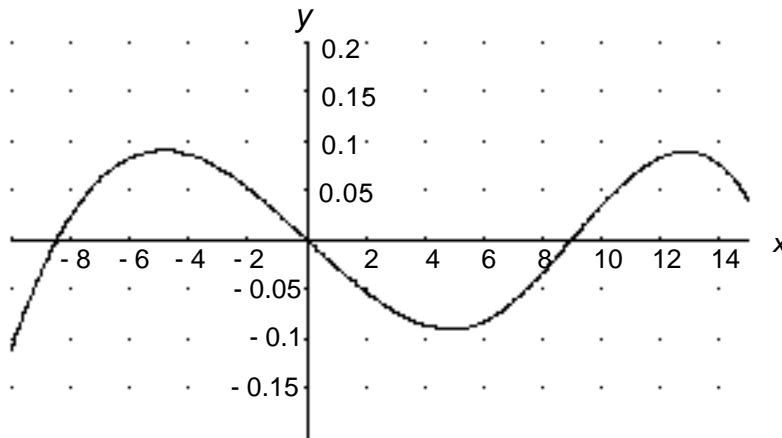
Note: Derivation of the formulas for $M(x)$ requires developing the concept of internal forces and moments as well as a method that involves sectioning a beam. Further information can be found in the reference at the end of the Spinoff.

Part I: Singularity Functions

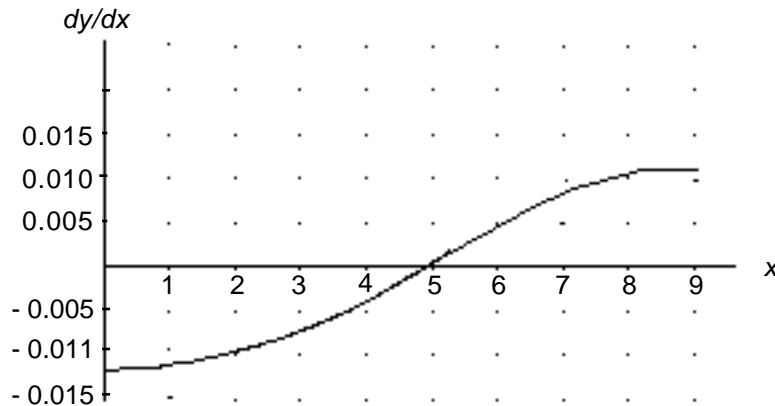
- 1) On the coordinate axes provided below, sketch the graphs of $\langle x - a \rangle^0$, $\langle x - a \rangle^1$, and $\langle x - a \rangle^2$, for $a > 0$.



- 2) Using Equation 2, sketch the graph of $y(x)$ on $[-10, 15]$. Use feet for the unit on the x -axis and inches for the unit on the y -axis.



- 3) In the derivation of a formula for the deflection, $y(x)$, the approximation $1 + \frac{dy}{dx}^2$ was made. Use your calculator/software to verify that this approximation is reasonable by graphing $\frac{dy}{dx}$ on $[0, 9]$. Then, calculate $1 + \frac{dy}{dx}^2$ for the largest value and compare it to 1.



largest value of $1 + \frac{dy}{dx}^2$: ~1.0001164

- 4) The function $y(x)$ in Equation 2 contains a singularity function. Is $y(x)$ differentiable on $(0, 9)$? Explain your answer.

Since this exercise involves the derivative of a piecewise polynomial function, the students only need to check $x = 6$. DERIVE works nicely here for doing left- and right-hand limits to show that $y(x)$ is differentiable on its domain. They can also show that $y \rightarrow 0$ as $x \rightarrow 0$.

Part II: Unit Analysis

- 1) In the sample calculation it was stated that the unit of the terms on the right-hand side of Equation 2 was lb-ft^3 . Explain why the unit is lb-ft^3 . Hint: $M(x)$ was integrated twice to obtain $y(x)$.

Students should be able to explain that the units on an integral, $\int f(x)dx$, are the product of the units on $f(x)$ and x .

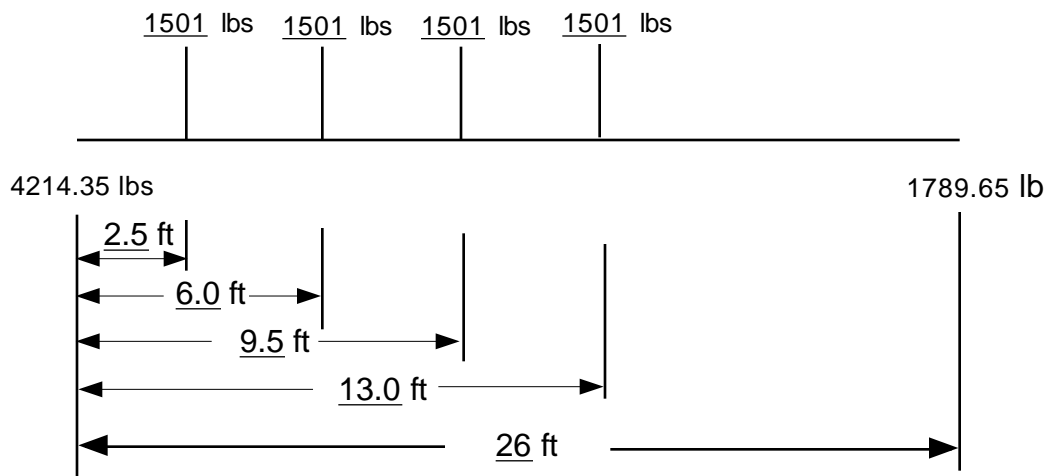
- 2) In the sample calculation, the numerical value of C_1 turned out to be - 600. What is the unit that needs to be assigned to C_1 in order for Equation 2 to make sense? Explain your reasoning.

The product C_1x must have the unit lb-ft^3 in order to have like terms on the right-hand side of equation 2. So, C_1 must be in lb-ft^2 .

Part III: Calculating $y(x)$

Referring to the floor plan on the last page of Spinoff 2B, the third row of aquaria from the right-hand side consists of four racks of 50 gallon aquaria. The aquaria are stacked three high in a rack. Water weighs 8.34 lb./gal., and each rack which holds three aquaria weighs 250 lb. As a convenient approximation, suppose that the combined weight of water and rack is concentrated at the geometric center of the rectangles shown in the floor plan.

- 1) Assume the row of 50 gallon aquaria rests on a reinforced concrete beam 26 feet long. Use a ruler to estimate to the nearest 1/16 inch (i.e., the nearest 1/2 foot), the distance from the wall at the top of the page to the geometric center of each rectangular rack. (Use the outside of the double line in the floor plan as the end of the beam.) Fill in the distances and weights on the static diagram shown below. Also, calculate the reactive forces at the beam ends as outlined in the sample calculation, and fill in the reactive force values at the beam ends in the following diagram.



- 2) Using the data from the diagram, calculate $y(x)$ for this situation. You will have four singularity functions in your answer. Also, state the boundary conditions which are needed to determine the constants of integration.

Boundary conditions: $y(0) = y(26) = \underline{0}$

$$EIy(x) = \frac{4214.35}{6} x^3 - \frac{1501}{6} [\langle x - 2.5 \rangle^3 + \langle x - 6 \rangle^3 + \langle x - 9.5 \rangle^3 + \langle x - 13 \rangle^3] - 208611x$$

Before proceeding, verify your solution above with at least one other group or your instructor.

- 3) For the reinforced concrete beam, use $E = 6 \times 10^6$ lb/in² and $I = 700$ in⁴. Differentiate $y(x)$ and use the result to calculate the location and the amount of the maximum deflection of the beam.

x-value (location): 12.196 ft

Maximum deflection: 0.643 in

The values of E , the modulus of elasticity, and I , the moment of inertia, used above are ballpark estimates. A rule of thumb for structural engineering is that the maximum deflection should not be more than the length of the beam in inches divided by 360. Here, $12(26)/360 = 0.867$, so the deflection above seems to be “in the ballpark.” However, the weight of the beam has not been taken into account in the function $y(x)$.

Part IV: A Safety Problem

When NASA designers inspected the lab as equipment was being installed, there was an extra water tank, cylindrical in shape, 4 feet in diameter and 6 feet high, located near the outside wall at point A on the floor plan. The purpose of the extra tank was to cool outside water and provide a source of “settled” water for the aquaria and related experiments. Unfortunately, the tank capacity was far too large for the reinforced concrete floor on which the tank was standing. From original blueprints and building specifications, a NASA engineer calculated that the floor could safely support no more than 200 lb/sq ft.

Calculate the distributed load of the full tank sitting on the floor with no special support, and write a concluding statement which indicates a reasonable and simple solution to the problem caused by the presence of the extra tank.

A full tank would exert 374.4 lb per ft² on the floor - well above the limitation indicated by the NASA engineer.

REFERENCE:

Beer and Johnston, Jr.: Mechanics of Materials, 2nd ed., Section 7.5, pp. 432-434, McGraw-Hill, Inc. 1992.