LTA 16

NASA - AMATYC - NSF
Project Coalition

Kennedy Space Center

The Space Shuttle Landing

Mathematics for Engineering Technology

Aeronautical
Space

Capital Community College

An aerial view of the Shuttle Landing Facility Runway as a Space Shuttle makes its approach.

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Mathematics for
Aeronautical Engineering Technology
Space Engineering Technology

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Figure 1: Space Shuttle Orbiter Returns

Introduction

Space Shuttle missions have become a familiar topic on the evening news and undoubtedly the fiery launch of a Shuttle from the Kennedy Space Center is a marvel to behold. However, the final and most critical phases of a Space Shuttle mission are deorbit, re-entry into the atmosphere, and landing. This LTA will focus on the final landing approach of the Shuttle, but first here is a brief description of the events from deorbit until final approach.

The Space Shuttle has no power and essentially acts as a glider, although it is much heavier and has a less aerodynamic shape than an actual glider (it has been referred to as a "flying brick"). It has only one chance to land since, without engines, it cannot climb and try another approach. The process begins half a world away from the landing runway, when the Space Shuttle is traveling 200 miles above the ground at a speed of over 17,000 miles per hour. It is now about 60 minutes to touchdown. During the deorbit burn, the Space Shuttle travels tail first and loses some speed and altitude. Once the burn is complete the Space Shuttle is reversed, its nose is raised, and the atmospheric entry begins. It is now about 31 minutes to touchdown. During this phase, there is a tremendous heat buildup around the Shuttle and portions of the vehicle's exterior reach 2,800°F (you have probably heard about the tiles used on the surface of the vehicle to protect it at this critical time). The heat strips electrons from the air around the Space Shuttle, enveloping it in a sheath of ionized air that blocks all communication with the ground for about 12 minutes. During this interval the pilot will perform several banking maneuvers called roll reversals or S-turns to control descent. When the Space Shuttle comes out of the communications blackout, its speed is about 8,275 mph and 12 minutes remain to touchdown. It is now committed to a particular landing site and must begin the final approach with enough altitude and speed to reach the touchdown point. At this point the vehicle travels a circular path around an imaginary cone that will line it up with the center line of the runway. Once the Shuttle comes out of this turn, it is ready for its final approach to the runway.

In this activity, you will examine the path that the Shuttle takes during its final approach to the runway.

Version I

The Final Approach to the Runway
Using Linear Functions, Exponential Functions, and Trigonometry

Coming out of its turn, the Shuttle should be at an altitude of 13,365 ft, have a speed of 424 mph, and be 7.5 miles (horizontal distance) from the runway. It is now 86 sec to touchdown. The nose is down so that the Space Shuttle can descend steeply to a point 7,500 ft from the runway threshold, where its altitude should be 1,750 ft. The vehicle then enters a transitional phase. The Shuttle's nose is raised as it heads for a position where its altitude is 131 ft and its distance from the runway threshold is 2,650 ft. The Shuttle is now 17 sec to touchdown. From here the Space Shuttle enters the final phase, aiming at a point 2,200 ft down the runway.

1) To begin to examine the **path** of the Shuttle on its final approach to the runway, set up a coordinate system and plot the four data points (from the paragraph above) that mark the location of the Shuttle. (Do not connect the points at this time.) Note that the horizontal distances are measured from the runway threshold (the beginning of the runway), but one measurement is in the opposite direction from the other three. Also, the units of measurement are not the same. When you construct your coordinate system, use the runway threshold as your origin. Pay careful attention to the scale on each axis and use consistent units of measure. Select vertical and horizontal scales that allow you to clearly see each phase of the Shuttle's flight path. You may need to paste several sheets of graph paper together. Attach word labels to the axes that include the unit of measure used. Take your time because a complete and accurate sketch will give you a better understanding of the situation.

2) Are your data points rising or falling? Is the Shuttle going up or coming down? Do your answers to both questions have to be the same? Why or why not?

Now we will look at each phase of this final approach to the runway and examine it more carefully.

NOTE: Rounding directions are given throughout. Be sure to follow directions.

The First Phase

The first phase is the initial steep descent of the Shuttle, which takes it from the first point, where the altitude is 13,365 ft to the second point, where the altitude is 1,750 ft. Refer to the previous description of the final approach to the runway. This phase is best modeled by a linear function.

3) Find the slope of the line segment containing the two data points. Round to 5 decimal places. Remember that the distance from the runway threshold is input and the altitude is output. When you compute the slope, what are the units of the numbers used in the ratio? Since a slope is a rate which compares two quantities, explain what information the slope gives you.

4) Use the slope to write a linear function to fit these two data points. Round all decimals to 5 places. For what values of the input variable is this function a valid model? This is the domain of the linear function.

5) Draw the graph of your function between the two data points. Label the graph with its equation.

6) Use your model to predict the altitude of the Shuttle when it is 10,000 ft from the runway threshold. Round to the nearest whole foot.

7) The slope of this line segment gives you some idea of the steepness of descent of the Shuttle during this phase, but NASA flight engineers prefer to use *glide slope*. Glide slope is the angle, measured in degrees, which the line makes with the horizontal. Use trigonometry to find the glide slope during this phase. Round to the nearest tenth of a degree. Note: Be sure your calculator is in degree mode.

   Note: For comparison, the glide slope used by commercial aircraft when landing is approximately $3°$.

The Transitional Phase

The transitional phase takes the Shuttle from its steep linear descent path to its final shallow linear path. During this time, the vehicle shifts from nose down to nose up. This phase begins at the point where the altitude is 1,750 ft and ends at the point where the altitude is 131 ft. We will first try a linear model.

8) Find the linear function that contains the two data points for the transitional phase. Round all decimals to 5 places.

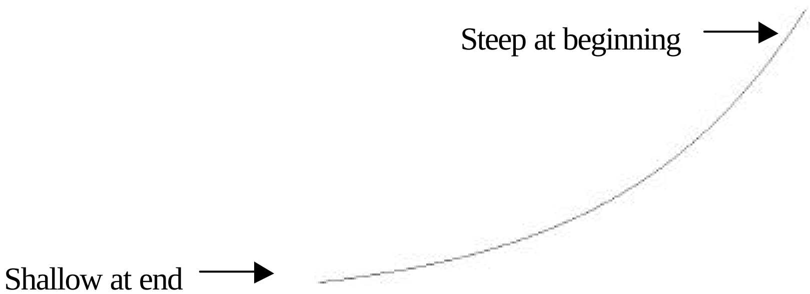
9) Find the glide slope for this linear model. Round to the nearest tenth of a degree.

10) Does the answer you just obtained trouble you? The Shuttle should enter the transitional phase at a steep glide slope but come out of it at a shallow glide slope. Could this ever happen with a linear model? Why or why not?

11) On your answer paper (but not on the graph), sketch a path shape that might accomplish the transitional phase.

The Exponential Capture

The linear model for the transitional phase is not successful; that is, a straight line path cannot be steep at the beginning and shallow at the end. Your sketch of a more appropriate path might look like this:



To model this shape, an exponential function is a reasonable choice since exponential functions have a very flat part and a very steep part. For this reason, we will now try an exponential model.

12) We will assume that the equation of this exponential function is of the form $y = ab^x$. Substitute the coordinates of each point for the transitional phase into this equation. This will give you two equations in *a* and *b*. Write down these two equations.

13) Solve this system of equations to find the values of *a* and *b* and use your values of *a* and *b* to write the exponential function. For *b*, use all the decimal values given by your calculator. For *a*, round to 5 decimal places. This exponential function is sometimes called the "exponential capture" function because it "captures" the glide slopes on either side. What is the domain of the exponential function in this situation?

14) You can use a graphing calculator to check your answer. On a TI-82™ or TI-83™, press STAT, then EDIT. Enter your two *x*-values into list L_1 and enter your two *y*-values into list L_2. Press STAT again, then CALC and choose ExpReg. Press ENTER at the home screen and you will get the exponential function that passes through your two points. It should agree fairly well with the equation you found.

15) On your large graph, draw the graph of your function between the two data points. Label the graph with its equation.

The Final Phase

The final phase is the shallow linear path that takes the Shuttle to its landing position. It starts at the point where the altitude was 131 ft and ends at touchdown 2,200 ft past the runway threshold.

16) Write a linear function to fit the two data points for the final phase. Round all decimals to 5 places. What is the domain of this function for this situation?

17) On your large graph, draw the graph of your function between the two data points. Label the graph with its equation.

18) Find the glide slope for this linear model. Round to the nearest tenth of a degree.

A Comparison

19) Recall that commercial aircraft use an approximate 3° glide slope for their runway approach. Suppose a commercial plane began its runway approach from the same initial altitude as the Shuttle (13,365 feet). Draw a triangle and use trigonometry to find the horizontal distance needed to land. Give your answer in both feet and miles, rounded to the nearest whole number. For comparison, compute the total horizontal distance covered by the Shuttle in both feet and miles.

Note: If you plot all these graphs carefully on your graphing calculator, you may notice that where the exponential curve meets the line of the first phase there is a fairly sharp "corner". In reality the Space Shuttle cannot make instantaneous changes in its direction. As a result, the Shuttle must follow a path at these boundary points that smooths out the curve. If you have a background in calculus, this issue is discussed further in Spinoff 16A and Spinoff 16B.

Summary

20) In this exercise, you should work to produce a summary of your work. Your intended audience is a student at your level of mathematical experience, and your intention is to provide a mathematical glimpse of the final runway approach of the Space Shuttle. Use your graph and describe each of the three phases of the landing path. Discuss the transitional phase and explain why an exponential function is a better model than a linear one. Conclude with the equations you found, written as a piecewise function. Note: If you have no experience with piecewise functions and graphs, please see Part B of Version II.

How Does It Actually Work?

You might be wondering at this point whether the Shuttle pilot does rapid mathematical equations in his or her head as the Shuttle is descending! Of course, this is ridiculous, so there must be a way for the pilot to keep the Shuttle on the path determined by these equations without doing any calculations. Naturally, there are onboard computers that help with navigation, but there are also visual aids for manual landing. A set of lights, called Precision Approach Path Indicator (PAPI) lights provide a visual indication of the correct glide slope for the first phase of the landing. This glide slope is referred to as the *outer glide slope*. These lights are at 7,500 feet from the runway threshold. White lights are seen by the crew if the Shuttle is above the glide slope; red lights show if they are below the glide slope. If they are on the correct glide slope, both red and white lights can be seen. The equations that you found were used to determine the correct placement and angle of the lights.

The glide slope during the final phase is called the *inner glide slope* and the Ball-Bar light system is a visual reference to show the pilot the correct inner glide slope. The bar lights are red lamps in a horizontal row. They are 2,200 ft beyond the runway threshold. In front of the bar lights is a white "ball" light. If the Shuttle is above the glide slope for the final approach, the "ball" will appear to be below the "bar"; if it is below the glide slope, the "ball" will appear to be above the "bar". If the glide slope is correct, the "ball" will appear to sit right on the "bar" and as the Shuttle moves closer the ball lights will appear to move outward along the bar. Once again, the equations you found were used to determine the correct placement of these lights.

The exponential capture function happens by a "preflare" maneuver. During this maneuver, the nose of the Shuttle is raised. This allows the Shuttle to gradually shift from the outer glide slope to the inner glide slope through the exponential path.

Version II

The Final Approach to the Runway Using Linear Functions

Coming out of its turn, the Shuttle should be at an altitude of 13,365 ft, have a speed of 424 mph, and be 7.5 miles (horizontal distance) from the runway. It is now 86 sec to touchdown. The nose is down so that the Space Shuttle can descend steeply to a point 7,500 ft from the runway threshold, where its altitude should be 1,750 ft. The vehicle then enters a transitional phase. The Shuttle's nose is raised as it heads for a position where its altitude is 131 ft and its distance from the runway threshold is 2,650 ft. The Shuttle is now 17 sec to touchdown. From here the Space Shuttle enters the final phase, aiming at a point 2,200 ft down the runway.

Part A Developing the Equations

1) To begin to examine the **path** of the Shuttle on this final approach to the runway, set up a coordinate system and plot the four data points (from the paragraph above) that mark the location of the Shuttle. (Do not connect the points at this time.) Note that the horizontal distances are measured from the runway threshold (the beginning of the runway), but one measurement is in the opposite direction from the other three. Also, the units of measurement are not the same. When you construct your coordinate system, use the runway threshold as your origin. Pay careful attention to the scale on each axis and use consistent units of measure. You want to be able to clearly see each phase of the Shuttle's flight path, so your vertical and horizontal scales should allow for this. You may need to paste several sheets of graph paper together. Attach word labels to the axes that include the unit of measure used. Take your time because a complete and accurate sketch will give you a better understanding of the situation.

2) Are your data points rising or falling? Is the Shuttle going up or coming down? Do your answers to both questions have to be the same? Why or why not?

Now we will look at each phase of this final approach to the runway and examine it more carefully. **NOTE:** Rounding directions are given throughout. Be sure to follow directions.

The First Phase

The first phase is the initial steep descent of the Shuttle, which takes it from the first point where the altitude is 13,365 ft to the second point where the altitude is 1,750 ft. Refer to the previous description of the final approach to the runway.

3) Put the two data points from the first phase into a table, using distance from runway threshold as input and altitude as output.

4) This phase is best modeled by a linear function. Find the slope of the line segment connecting the two data points. Round to 5 decimal places. When you compute the slope, what are the units of

the numbers used in the ratio? Since a slope is a rate, which compares two quantities, explain what information the slope gives you.

5) Use the slope to write the equation of the line through these two data points. Round all decimals to 5 places. For what values of the input variable is this function a valid model?

6) On your large graph, draw the graph of your line between the two data points. Label the graph with its equation.

7) Use your equation to predict the altitude of the Shuttle when it is 10,000 ft from the runway threshold. Round to the nearest whole foot.

The Transitional Phase

The transitional phase takes the Shuttle from its steep descent path to its final shallow path. This begins at the point where the altitude is 1,750 ft and ends at the point where the altitude is 131 ft.

8) Put the two data points from the transitional phase into a table, again using distance from runway as input and altitude as output.

9) Find the slope of the line segment connecting the two data points. Round to 5 decimal places.

10) Write the equation of the line connecting these data points. Round all decimals to 5 places. For what values of the input variable would this function be a valid model?

11) On your large graph, draw the graph of your line between the two data points. Label the graph with its equation.

12) Hopefully the answer you just obtained troubles you. The Shuttle should enter the transitional phase at a steep glide slope but come out of it at a shallow glide slope. Could this ever happen with a linear model? Why or why not? Sketch a path shape that might accomplish the transitional phase.

The Final Phase

The final phase is the shallow path that takes the Shuttle to its landing position. It starts at the point where the altitude was 131 ft and ends at touchdown 2,200 ft past the runway threshold.

13) Put the two data points from the final phase into a table, again using distance from runway as input and altitude as output.

14) Find the slope of the line segment connecting the two data points. Round to 5 decimal places.

15) Write the equation of the line connecting these two data points. Round all decimals to 5 places. For what values of the input variable is this function a valid model?

16) On your large graph, draw the graph of your line between the two data points. Label the graph with its equation.

Part B Compound Inequalities and Piecewise Graphs

Compound Inequalities

You now have three linear equations to use to describe the path taken by the Space Shuttle in its landing. However, each equation is used for only a certain set of x values. This set of x values is called an *interval*. For example, the equation you found for the first phase is used only when x is between 7,500 ft and 39,600 ft. If we want a complete and accurate mathematical description of the path during each phase, we need to include the x-interval for each equation. The mathematical notation to describe an interval is a *compound inequality.* Suppose we wanted to say that x is a number between 50 and 2,000, not including either number. The compound inequality is $50 < x$ AND $x < 2000$. It means that $50 < x$ AND $x < 2000$, but it is read "x is between 50 and 2,000". If 50 and 2,000 are included, the compound inequality is $50 \le x \le 2000$. If 50 is included but 2000 is not, the compound inequality is $50 \le x < 2000$.

17) Write a compound inequality for the statement: x is between 3,000 and 12,000, not including either number.

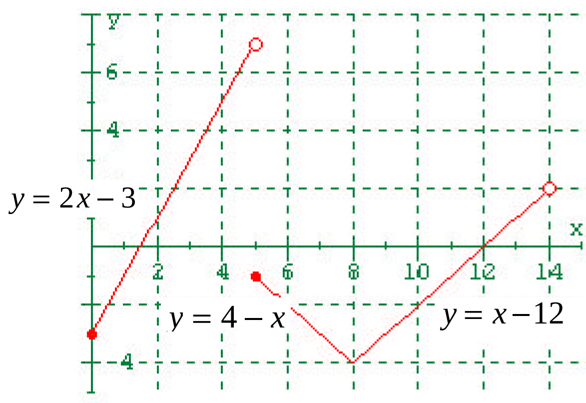
18) Write a compound inequality for the statement: x is between 120 and 765, including both numbers.

Piecewise Graphs

When a graph is composed of several pieces, with each piece having its own equation and x-interval, that graph is called a *piecewise graph*. If each piece is linear, the graph is called a *piecewise linear* graph. Here is an example:

$$y = 2x - 3 \quad \text{if } 0 \le x < 5$$
$$y = 4 - x \quad \text{if } 5 \le x \le 8$$
$$y = x - 12 \quad \text{if } 8 < x < 14$$

This means that the line $y = 2x - 3$ is used *only* when x is between 0 and 5 (including 0 but not 5), the line $y = 4 - x$ is used *only* when x is between 5 and 8 (including both), and the line $y = x - 12$ is used *only* when x is between 8 and 14 (not including either number). This is what the graph would look like:



Notice that the first and second "piece" do not connect. Since the endpoint 5 is not used for the first piece, there is an open dot on the graph of the first piece when $x = 5$. Since the endpoint 5 *is* included for the second piece, there is a closed dot on the graph of the second piece when $x = 5$. Since the second and third pieces do connect at the point where $x = 8$, there is no need for any kind of special dot there.

19) Suppose a piecewise linear graph is given by these equations:

$y = 3x + 2$ if $0 \le x \le 3$, and

$y = -x + 14$ if $3 < x < 7$

Explain in words what this set of equations means.

20) Graph this piecewise linear graph on graph paper. Remember that this is just one graph, but it has two pieces; each piece is used only for its identified x-interval.

21) Find y if $x = 1$. Clearly identify which piece is used.

22) Find y if $x = 6$. Clearly identify which piece is used.

The Shuttle Landing Equations

23) Write the equation for the first phase of the Shuttle's final runway approach and include the x-interval with the equation. Include both end numbers in your compound inequality. This equation should be labeled Equation 1.

24) Write the equation for the transitional phase of the Shuttle's final runway approach and include the x-interval with the equation. Exclude both numbers in your compound inequality. This equation should be labeled Equation 2.

25) Write the equation for the final approach and include the x-interval with the equation. Include both numbers in your compound inequality. This equation should be labeled Equation 3.

Using the Equations

26) Suppose the Shuttle is 5 miles from the runway threshold. Find its altitude to the nearest whole foot. Clearly identify which of your equations you used to obtain your answer.

27) Suppose the Shuttle is 1,000 ft from the runway. Find its altitude to the nearest whole foot. Clearly identify which of your equations you used to obtain your answer.

28) Suppose the Shuttle is 5,000 ft from the runway. Find its altitude to the nearest whole foot. Clearly identify which of your equations you used to obtain your answer.

29) Suppose the Shuttle is at an altitude of 1,450 ft. Find its distance from the runway threshold to the nearest whole foot. You will need to check your tables or the graph to decide which equation to use. Clearly identify which of your equations you used.

30) Suppose the Shuttle is at an altitude of 100 ft. Find its distance from the runway threshold to the nearest whole foot. Clearly identify which of your equations you used to obtain your answer.

Summary

31) In this exercise, you should work to produce a summary of your work on modeling the Shuttle landing. Your intended audience is a student at your level of mathematical experience and your intention is to provide a mathematical glimpse of the final runway approach of the Space Shuttle. Use your piecewise graph, and describe each phase of the landing path. Discuss the transitional phase, giving reasons why the straight line model is probably not correct. Conclude with the equations you found, along with the appropriate intervals.

How Does It Actually Work?

You might be wondering at this point whether the Shuttle pilot does rapid mathematical equations in his or her head as the Shuttle is descending! Of course, this is ridiculous, so there must be a way for the pilot to keep the Shuttle on the path determined by these equations without doing any calculations. Naturally, there are onboard computers that help with navigation, but there are also visual aids for manual landing. A set of lights, called Precision Approach Path Indicator (PAPI) lights provide a visual indication of the correct slope for the first phase of the landing. This slope is called the *outer glide slope*. These lights are at 7,500 feet from the runway threshold. White lights are seen by the crew if the Shuttle is above the correct glide slope; red lights show if they are below the glide slope. If they are on the correct glide slope, both red and white lights can be seen. The equations that you found were used to determine the correct placement and angle of the lights.

The slope during the final phase is called the *inner glide slope* and the Ball-Bar light system is a visual reference to show the pilot the correct inner glide slope. The bar lights are red lamps in a horizontal row. They are 2,200 ft beyond the runway threshold. In front of the bar lights is a white "ball" light. If the Shuttle is above the glide slope for the final approach, the "ball" will appear to be below the "bar"; if it is below the glide slope, the "ball" will appear to be above the "bar". If the glide slope is correct, the "ball" will appear to sit right on the "bar" and as the Shuttle moves closer the ball lights will appear to move outward along the bar. Once again, the equations you found were used to determine the correct placement of these lights.

The transitional phase happens by a "preflare" maneuver. During this maneuver, the nose of the Shuttle is raised. This allows the Shuttle to gradually shift from the outer glide slope to the inner glide slope. As you probably guessed, the actual path is not linear but is curved, like this: