SPINOFFS

Spinoffs are relatively short learning modules inspired by the LTAs. They can be easily implemented to support student learning in courses ranging from prealgebra through calculus. The Spinoffs typically give students an opportunity to use mathematics in a real world context.

LTA - SPINOFF 15A	The Capture-Recapture Method
LTA - SPINOFF 15B	Florida Scrub-Jay Populations and Habitat
LTA - SPINOFF 15C	Population Models with Recursive Equations

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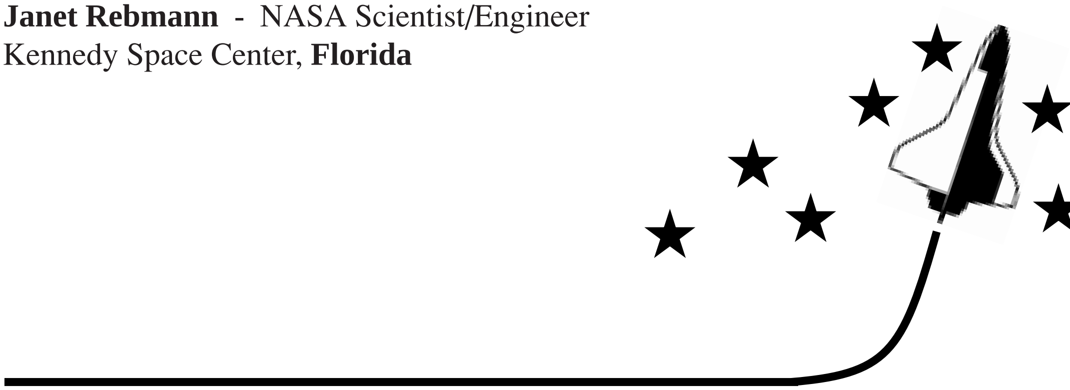

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SPINOFF 15C

Population Models with Recursive Equations

Mathematical models are important tools for monitoring and forecasting future population sizes. Four key factors affecting population size are:

- Birth rate
- Death rate
- Immigration rate
- Emigration rate

These components are affected by habitat conditions such as human activity, fire, predators, drought, vegetation, etc. In the NASA Kennedy Space Center surroundings, long-term effects include the environmental impact of "multiple launches and continuing land use changes." (http://atlas.ksc.nasa.gov/program.html).

Mathematical models incorporating all relevant components are extremely complex, but useful models may be constructed by including the four key factors listed above. Because births and immigration increase the population size, and deaths and emigration decrease it, a simple model can be described as follows:

New Population Size = (Previous Population Size) + (Births) – (Deaths) + (Immigration) – (Emigration).

Each term in this equation is based on a given time interval (a day, a year, etc.). We can represent the above terms with the following mathematical symbols:

N_{t+1} = New population size (population size at time t + 1)
N_t = Previous population size (population size at time t)
B = Number of births in one time unit
D = Number of deaths in one time unit
I = Number of new immigrants entering the population in one time unit
E = Number of emigrants leaving the population in one time unit

Using these symbols, we can rewrite the equation as follows: $N_{t+1} = N_t + B - D + I - E$

In a closed system we assume that there is no migration into or out of the system so that the equation becomes: $N_{t+1} = N_t + B - D$

In this model we may define the rate of growth as $r = (B - D)/N_0$, where N_0 is the population at time $t = 0$. As a result, we assume a constant rate of growth during the time interval. If the number of births

B exceeds the number of deaths D, then r > 0 and we conclude that the population is increasing. If the number of births B is less than the number of deaths D, then r < 0

and we conclude that the population is decreasing. If the number of births and deaths is equal (B = D), then r = 0 and we conclude that the population is in equilibrium or steady state.

Example

Consider a colony of mice which has an initial population size of $N_0 = 1000$ and for which the number of births in one year is 10% of the population size, while the number of deaths in a year is 4% of the population size. Since the number of births in a year exceeds the number of deaths it follows that the population is increasing. In this example the rate of increase is given by

$$r = \frac{B - D}{N_0} = \frac{100 - 40}{1000} = .06 = 6\%$$

The following table shows the previous population size, the number of births, the number of deaths, and the new population size for 5 years:

Table 1

t	N_t	B	D	B - D	N_{t+1}
0	1000	100	40	60	1060
1	1060	106	42	64	1124
2	1124	112	45	67	1191
3	1191	119	48	71	1262
4	1262	126	50	76	1338

The annual rate of growth is expressed as a percentage of the population size, so that the increase in population size is rN_t and the new population size is $(1 + r)N_t$.

It follows that

$$N_1 = N_0 + rN_0 = (1 + r)N_0$$
$$N_2 = N_1 + rN_1 = (1 + r)N_1 = (1 + r)^2 N_0$$
$$N_3 = N_2 + rN_2 = (1 + r)N_2 = (1 + r)^2 N_1 = (1 + r)^3 N_0$$

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$$N_t = N_{t-1} + rN_{t-1} = \ldots = (1 + r)^t N_0$$

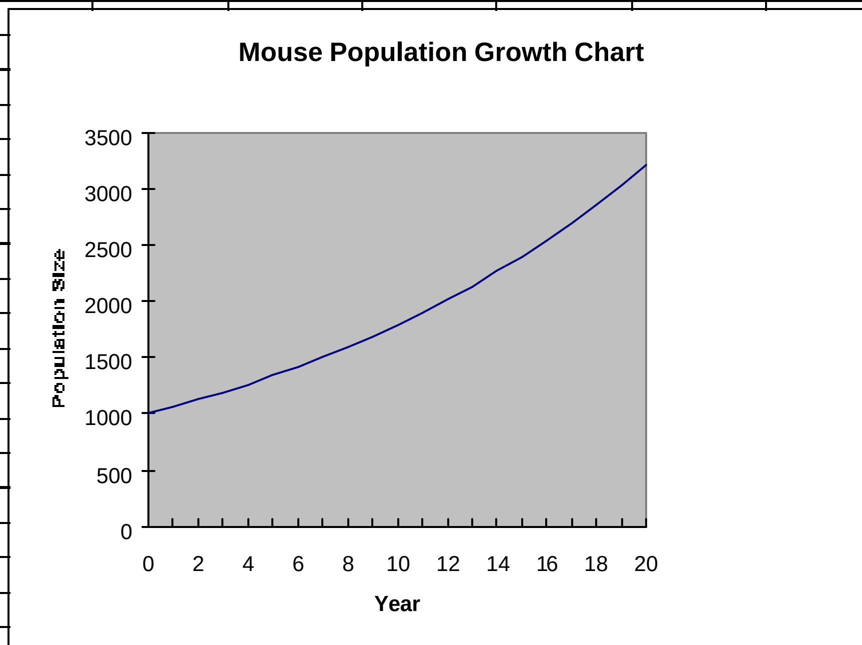
With r = 0.06 the formula becomes

$$N_t = 1000(1.06^t)$$

This formula is an example of exponential growth. Using EXCEL we can easily create a table representing 20 years of population growth (see Table 2). The accompanying graph is typical of exponential growth.

Table 2

Year	Population
0	1000
1	1060
2	1124
3	1191
4	1262
5	1338
6	1419
7	1504
8	1594
9	1689
10	1791
11	1898
12	2012
13	2133
14	2261
15	2397
16	2540
17	2693
18	2854
19	3026
20	3207



Exercises

Consider a population of birds which is threatened due to damage to its natural habitat. At the beginning of 1985 the population was estimated to be 5800 birds. The annual birth rate is estimated to be 3% of the population, and the death rate is estimated to be 8% of the population.

1) Determine an exponential growth formula that yields the population size at any time t.

2) Create a table of population sizes from the beginning of 1985 to the beginning of 2000.

3) Draw a graph representing the bird population from 1985 to 2000.

4) Estimate the population size at the beginning of 2010.

5) In what year will the population be one half of its size at the beginning of 1985?

6) How long will it take for the population to become extinct?