SPINOFFS

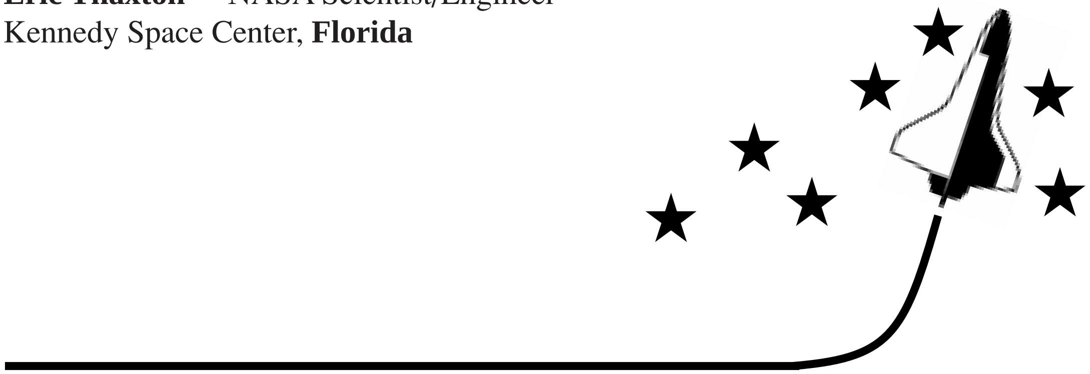
Spinoffs are relatively short learning modules inspired by the LTAs. They can be easily implemented to support student learning in courses ranging from prealgebra through calculus. The Spinoffs typically give students an opportunity to use mathematics in a real world context.

LTA - SPINOFF 14A	NASA Launch Vehicle Calculations
LTA - SPINOFF 14B	Force Versus Displacement
LTA - SPINOFF 14C	Analyzing Graphs

Jerry Keepers - AMATYC Writing Team Member
Potomac State College, Keyser, **West Virginia**

Cyrus McCarter - AMATYC Writing Team Member
Wake Technical Community College, Raleigh, **North Carolina**

Eric Thaxton - NASA Scientist/Engineer
Kennedy Space Center, **Florida**



Project Grant Team

John S. Pazdar
Project Director
Capital Community College
Hartford, Connecticut

Patricia L. Hirschy
Principal Investigator
Asnuntuck Community College
Enfield, Connecticut

Peter A. Wursthorn
Principal Investigator
Capital Community College
Hartford, Connecticut

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SPINOFF 14C

Analyzing Graphs

Background

Space vehicles are expensive to launch. The rockets, which are used to lift a vehicle into space, are only able to lift a certain maximum weight. Since both the vehicle's structure and payload contribute to the total weight, any decrease in structural weight would increase the payload weight that could be launched. As a result, it is important to design a vehicle and launch system so that the vehicle’s structural weight is kept to a minimum. It is also important, where possible, to keep external forces on the vehicle to a minimum. External forces include the force of winds in the atmosphere and the jerking motion encountered when the bolts, which hold the rocket in place before launch, are blown loose. If the external forces can be reduced, then the vehicle’s structure will not need to be as strong. This in turn will allow the structural weight to be reduced. In summary, to maximize the payload weight which is typically less than 10 percent of the total vehicle weight at liftoff, it is necessary to minimize the external forces encountered during launch and flight.

Just before liftoff, the rocket engines are ignited and they build up thrust to full power in about 4.5 seconds. During the buildup, the engines ignite at different times, so the thrust is uneven. If the vehicle were just resting on the launch pad, it would tilt and fall over while the engines were building up thrust. Therefore, the vehicle must be held down until all of the engines are balanced, working properly, and up to full power. When all the conditions are go, the vehicle is released.

Section I

Release Mechanisms

There are two general types of release mechanisms: soft and hard. A hard release is an instant release at full power. Typically, the hold down bolts of the restraining mechanism are secured by explosive nuts that are blown away to release the vehicle. A hold down bolt with an explosive nut is referred to as a pyrobolt. There is then a sudden jerk as the vehicle accelerates off the launch pad. This is analogous to what would happen at a tractor-pull contest if the cable attaching the tractor to the load suddenly broke. If the cable broke, the tractor would lurch ahead and the driver would be slammed against the back of the seat. Much more force is involved in launching a space vehicle. This sudden acceleration, or jerk, is hard on both payload and passengers. In order to mitigate the jerk, a controlled (soft) release mechanism, CRM, is added to the hard release structure. The diagram in Figure 1 on the next page shows the general components of a release mechanism and their relative positions. On the following page, Figure 2 shows how controlled release mechanisms are located relative to the rocket and the launchpad. If a soft release mechanism is used in a launch, there would be little or no jerk. Then, it is possible that the structural weight of the vehicle could be reduced and its payload capacity increased.

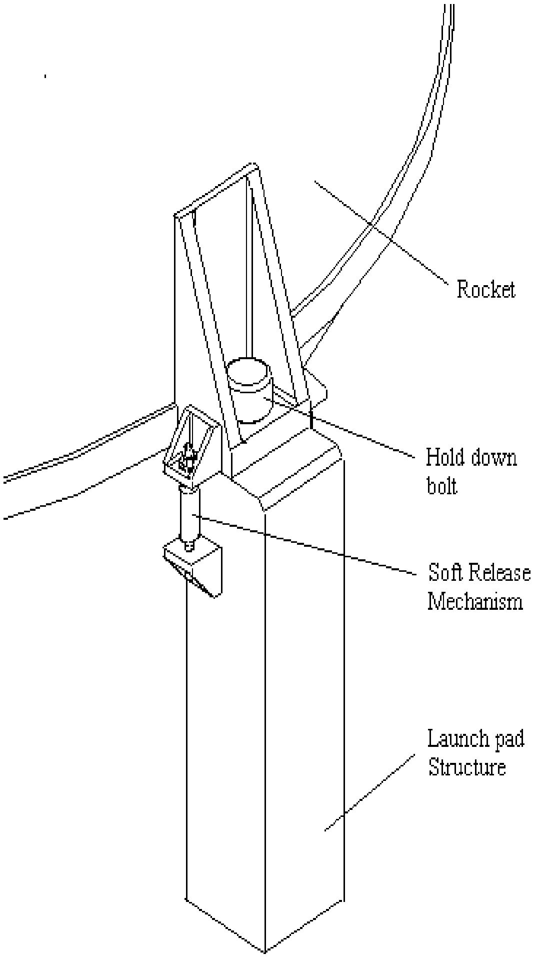

Figure 1: Controlled Release Mechanism

Figure 2 below shows controlled release mechanisms relative to a rocket and the launchpad.

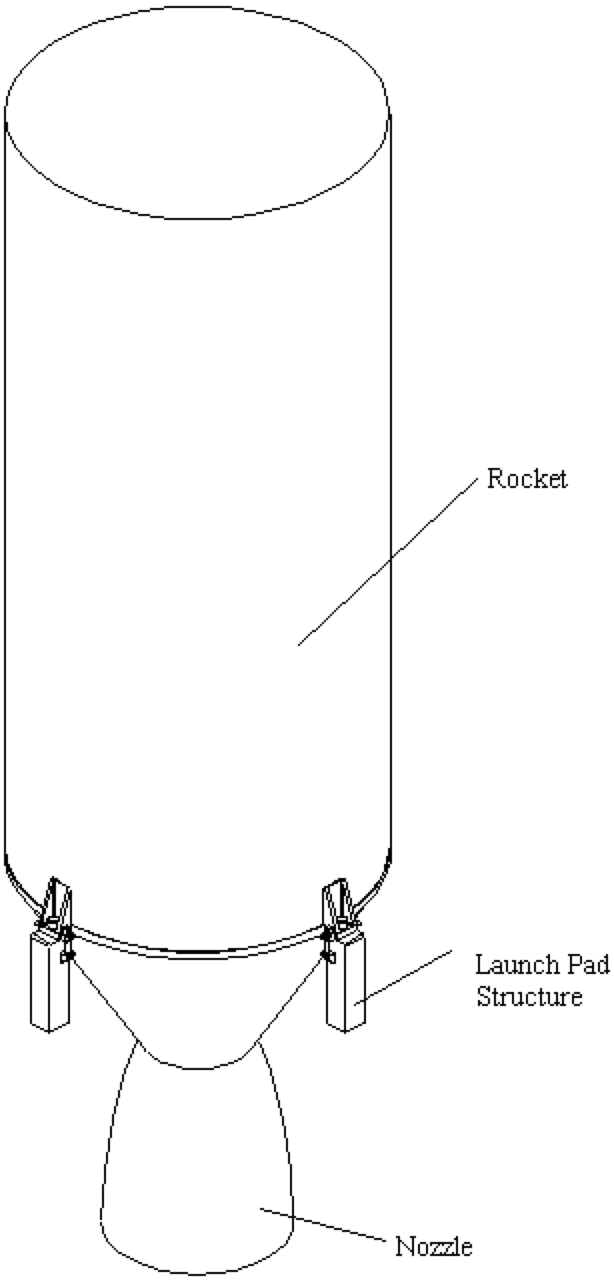

Figure 2 Hold down location on rocket structure

The new Evolved Expendable Launch Vehicle (EELV) is now on NASA's drawing boards. NASA scientists are presently evaluating several release mechanisms. To maximize the payload, the EELV will use eight controlled release mechanisms (CRMs), not just a simple hard release system. Figure 3 shows a simplified diagram of the launch forces acting on each CRM. The divisions by 8 shown in the diagram assume that the launch forces are distributed equally to each of the eight CRMs.

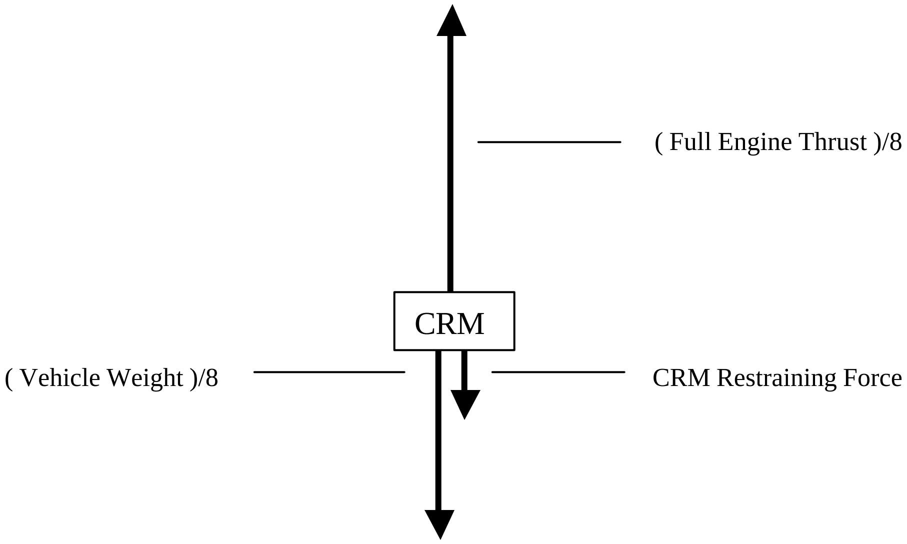

Figure 3: Launch Force Diagram

At the moment when the pyrobolts are blown away, the CRM restraining force together with one-eighth of the vehicle weight should equal the upward thrust of the rocket. The restraining forces of each CRM will diminish to zero, the **net** upward force will increase, and the vehicle will ascend.

Section II

Characteristics of a Restraining Force vs. Displacement Graph

The graphs of the force versus displacement functions in Figure 4 on the next page represent the same three forces that are shown above in Figure 3. These graphs pertain to any one of the eight CRMs and assume that the forces are equally distributed to each CRM. Notice that at zero (0) displacement, one-eighth of the vehicle's weight added to the restraining force equals one-eighth of the full thrust. The graph of the Restraining Force vs. Displacement function is typical of a good soft release mechanism.

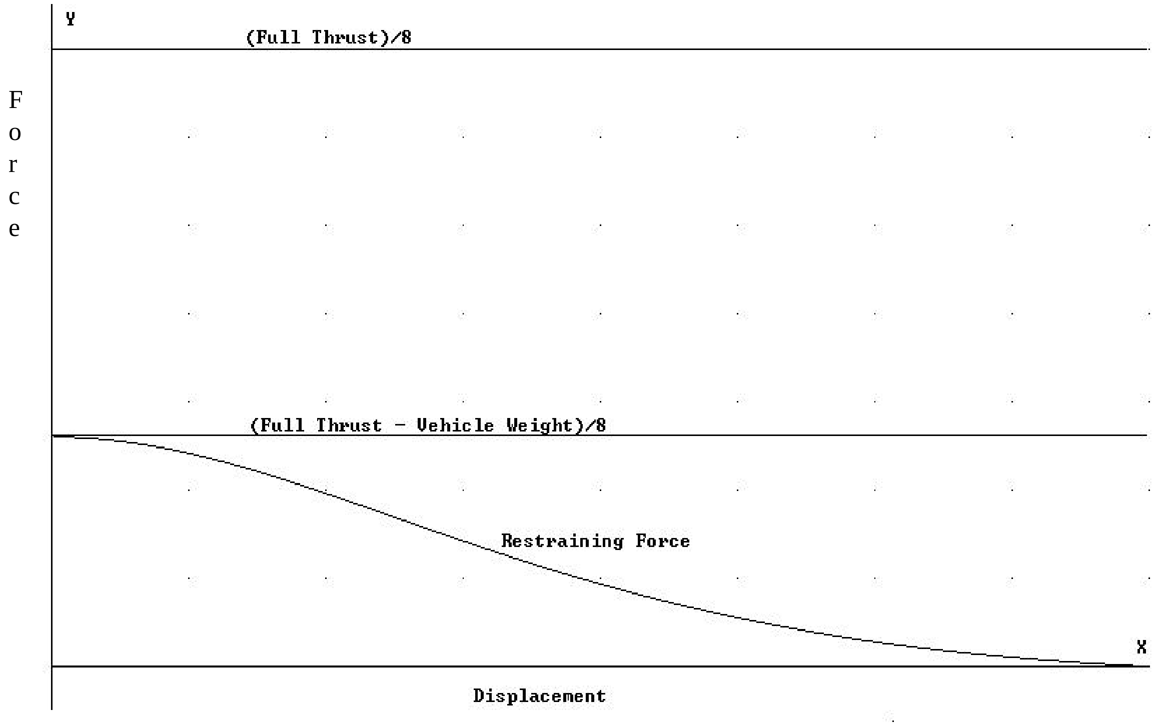

Figure 4: Launch Forces on Rocket

Important characteristics of a CRM restraining force graph:

- **Flat or nearly flat at the initial and final displacements:** This is significant because there will be little or no change in acceleration at these displacements. So, when the pyrobolts are blown away and the vehicle finally lifts clear, there will be a smooth transition. In general, sharp drops in the force curve are undesirable since these cause sudden jerks on the launch vehicle.

- **Decreasing restraining force:** According to Newton's Third Law, F = ma, the acceleration of the launch vehicle is directly proportional to the difference between the thrust of the rockets and the sum of the vehicle's weight and the combined restraining forces of the release mechanisms. The objective of the soft release mechanisms is to provide a smooth, yet swift, liftoff. If the force curve were to increase with displacement, then the acceleration of the vehicle would be impeded, slowing the launch of the vehicle and wasting energy. In fact, if the restraining force kept increasing indefinitely the vehicle would never leave the launch pad. Hence, the CRM restraining force should decrease throughout the entire displacement interval.

- **Low restraining force at release:** Upon release, if the restraining force is nonzero, there will be a jerk.

- **No sharp corners:** Sharp corners in the restraining force curve would also cause jerk.

Exercises

The exercises for this Spinoff refer to the following concepts for a soft release mechanism. For each concept, x represents the distance traveled off the launch pad (inches) and F_r is the restraining force (lbs).

Concept A

$$F_{r\,A}(x) = 50000 - 7143 \cdot x$$

Concept B

$$F_{r\,B}(x) = 50000 - 145.8 \cdot x^3$$

Concept C

$$F_{r\,C}(x) = 25000(1 + \cos(\frac{p \cdot x}{7}))$$

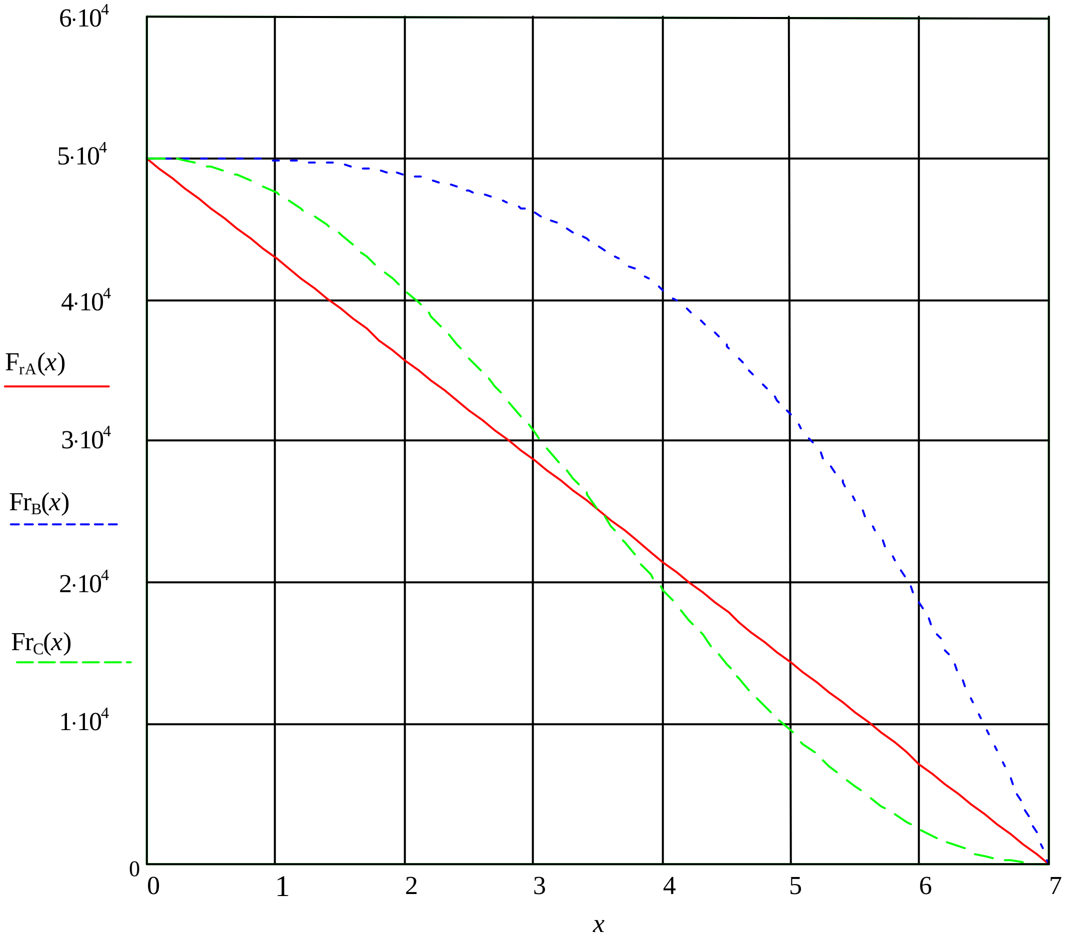

Figure 5: Force vs. Distance Relationships for Soft Release Mechanism Concepts

Exercises

1) Using the graphs of the force vs. distance relationships in Figure 4, what are the force values for each Concept (A, B, and C) at a distance, $x = 5$ inches?

2) At what distance(x) is the force for Concept A (F_{rA}) the same as for Concept C (F_{rC})?

3) A new soft release mechanism was proposed that has the following force-displacement equation: $F_{rD}(x) = 50{,}000\ e^{-x}$ where e equals 2.718 and is raised to the $-x$ power. The number, e, is the natural logarithm base (ln e = 1.0). Plot the new Concept D on the graph with concepts A, B, and C. Does it meet the requirement that the force equals 50,000 pounds at $x = 0$ and that at $x = 7$ inches the force equals zero within 100 pounds?

3) **Writing Assignment** Given the information provided about the graphs and properties of the CRMs, do the release mechanisms represented by Concepts A, B, C, D fit NASA's requirements for a CRM? Why or why not? Be sure to note critical characteristics each graph has or fails to have.

4) Hold Down Concept A costs $375,000 to design and test and $1,725 per hold down per launch to install. Concept B costs $1,025,000 to design and test and $975 per hold down per launch to install. (a) Which concept should NASA choose if there are 125 planned launches with eight hold downs used per launch? (b) Express the cost of each of the Concepts, A and B, in terms of a variable number of launches, x. Use these two functions to determine the "break even" number of launches. Solve this problem in two ways: graphically and algebraically.