SPINOFFS

Spinoffs are relatively short learning modules inspired by the LTAs. They can be easily implemented to support student learning in courses ranging from prealgebra through calculus. The Spinoffs typically give students an opportunity to use mathematics in a real world context.

LTA - SPINOFF 12A	Finding the Rate of Return for Energy Saving Investment
LTA - SPINOFF 12B	Fitting a Sine or Cosine Curve to NASA Energy Use Data
LTA - SPINOFF 12C	Rocket Spies: Codes and Rockets in the 1950’s and 1960’s

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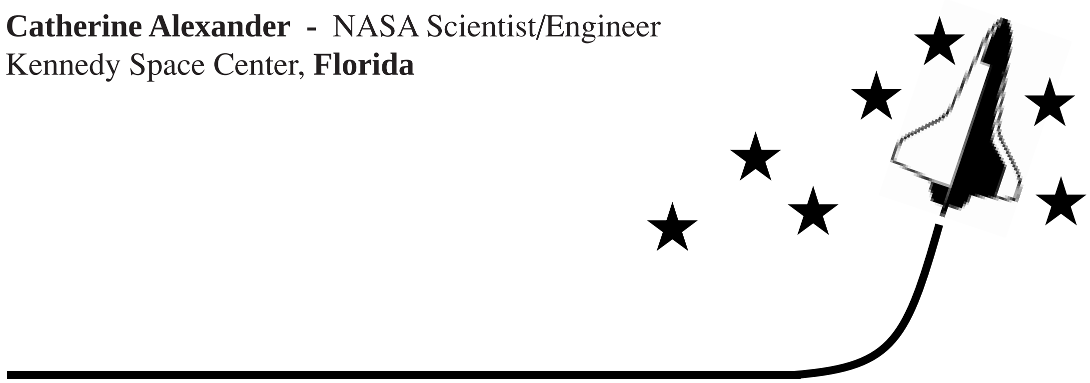

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SPINOFF 12A

Finding the Rate of Return for Energy Saving Investment

A private contractor has contracted with the government to modify 385 facilities located at Cape Canaveral Air Station (CCAS or the Cape) so that they will be more energy efficient. The contractor will invest $12,000,000 in energy modifications during fiscal years 1998, 1999, and 2000. The government will pay the contractor a total of $23,000,000 during the 13 years from 2000 to 2012. The $23 million pay back will be funded by savings resulting from reduced energy consumption and deferred maintenance.

From the government's point of view, the money repaid that exceeds the actual cost of the energy projects is interest or the cost of borrowing money. For the contractor, the money that exceeds the investment in the energy projects is a return on investment. When the interest and return are calculated as annual percents, they are known as the interest rate and the rate of return respectively. For any given project the numbers are the same except for point of view. The borrower (the government in this case) is concerned with interest, and the lender (the contractor in this case) is concerned with rate of return.

For Exercises 1 and 2, assume that the contractor's investments are made in three annual lump sums at the beginning of each fiscal year. The first investment is $3 million on October 1, 1997 (FY 1998). The second investment is $8 million on October 1, 1998 (FY 1999). The final investment is $1 million on October 1, 1999 (FY2000).

Assume that the government will pay back $23 million over the course of 13 years beginning in the fiscal year 2000. The pay back, $23 million, will be divided into 13 equal parts of $1,769,230.77 and paid once each fiscal year on September 30. Table 6 shows the government's payment schedule based on these assumptions.

Table 6 Government's Payment Schedule for Payback
Payments made annually on September 30th at the end of each fiscal year

Payment Dates	Amount Owed	Payment	Remaining Amount Owed
9/30FY00	23,000,000	1,769,230.77	21,230,769.23
9/30/FY01	21,230,769	1,769,230.77	19,461,538.46
9/30/FY02	19,461,538	1,769,230.77	17,692,307.69
9/30/FY03	17,692,308	1,769,230.77	15,923,076.92
9/30/FY04	15,923,077	1,769,230.77	14,153,846.15
9/30/FY05	14,153,846	1,769,230.77	12,384,615.38
9/30/FY06	12,384,615	1,769,230.77	10,615,384.61
9/30/FY07	10,615,385	1,769,230.77	8,846,153.84
9/30/FY08	8,846,154	1,769,230.77	7,076,923.07
9/30/FY09	7,076,923	1,769,230.77	5,307,692.30
9/30/FY10	5,307,692	1,769,230.77	3,538,461.53
9/30/FY11	3,538,462	1,769,230.77	1,769,230.76
9/30/FY12	1,769,231	1,769,230.77	-0.009999996

Exercises

Complete Exercises 1 and 2 to find the rate of return for the $12 million investment.

1) We know that the contractor made investments in each of three years that totalled $12,000,000. Assume a prevailing market interest rate of 7% per year compounded at the end of each fiscal year. Find the total value of the contractor's three investments on October, 1, 1999 (the beginning of FY2000). To do this, observe that on October 1, 1999, the contractor's first investment will have grown for two years, the second for one year, and the third for zero years.

2) If annual payments of $1,769,230.77 are made on September 30th for 13 years, find the rate of return if the end balance is to be 0 on September 30, 2012. The first payment is made on September 30, 2000. Use the value from Exercise 1 as the present value of the 13 payments that the contractor will receive from the government. You may solve this exercise by using a TI-83™ graphing calculator by following the steps shown on the next page.
Note: This repayment schedule can be viewed as an annuity of regular payments made at the end of annual payment periods. The term of the annuity begins on October 1, 1999, one payment period prior to the first payment, and ends on September 1, 2012, the date of the last payment.

The following steps may be used to find rate of return using the TI 83™ or the TI 83 Plus™ Graphing calculator:

Go to Finance mode (2nd x^-1) on the TI 83™ or “APPS” on the TI 83 Plus™
Press “ENTER” to select “1:TVM Solver…”
Fill in appropriate values for this problem:

N = 13
I% = 0 (Because we do not know the interest, the zero is acting as a place holder. This enables us to enter the remaining values.)
PV = 13904329 (This is our answer from Exercise 1.)
PMT = -1769230.77
FV = 0
P/Y = 1
C/Y = 1
PMT: END

We are now ready to solve for the unknown interest rate. Move the cursor to I% = 0. To access the command “SOLVE,” press the key “ALPHA” followed by the key “ENTER”.

Extended Exercises for Business Calculus

For Exercises 3 to 6, assume that the contractor’s investments will be made on a monthly basis at the beginning of each month during the fiscal year. For example, the first year’s investment of $3,000,000 will be spent in 12 equal installments of $250,000. However, we will still assume that the government makes its payments to the contractor at the end of each fiscal year.

3) What are the monthly installments for the second year’s $8,000,000 investment?

4) What are the monthly installments for the third year’s $1,000,000 investment?

5) If the contractor values money at 8% per year compounded monthly, what is the total value of the contractor’s investment at the end of three years?
Option 1: Use a calculator. Option 2: Use a spreadsheet.

6) The government will remit annual payments of $1,769,230.77 to the contractor for 13 years. The payments are made at the end of each fiscal year beginning on September 30, 2000 (end of FY00). Find the contractor’s annual rate of return if the end balance is to be $0.00 on September 30, 2012. **Hint**: The present value of this annuity is the total value of the contractor’s investments on October 1, 1999 (the beginning of FY00). From Exercise 5, you know the value of the contractor’s investments on September 30, 2000 (the end of FY00).