# LTA 11

*NASA - AMATYC - NSF*
*Project Coalition*

*Kennedy Space Center*

**Helium Usage at Kennedy Space Center**

*Mathematics for Engineering Technology*

**Chemical**
**Industrial and Management**
**Materials**

*Capital Community College*


The KSC Railroad System transports helium via tracks east of launch Pad 39A.

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## Helium Usage at Kennedy Space Center

*Mathematics for*
*Chemical Engineering Technology*
*Industrial and Management Engineering Technology*
*Materials Engineering Technology*

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# LTA 11

## Helium Usage at Kennedy Space Center

This LTA involves the mathematical relationships that describe the characteristics of a gas under different conditions of pressure, temperature and volume. The Kennedy Space Center (KSC) uses helium for a variety of activities, including purging and cleaning the hydrogen system and engines of the Space Shuttle Orbiters. The helium is delivered to KSC with a certain pressure, temperature and volume, but it is used under different conditions.

Following are some problems involving helium that could arise at KSC, and your task is to solve these problems. Resources called "Mathematical Asides" and Hints are set up as links to provide information that you may need to solve the problems or to do the exercises.

**Problem 1**

The Kennedy Space Center uses approximately 70,000 standard cubic feet (scf) of gaseous helium each day for normal operations and processing. Prior to the spring of 1998 KSC purchased its helium in gaseous form delivered in railcars. These special railcars consisted of a pressurized tube bank mounted on a railcar structure. To maximize deliverable helium volume, the tube banks were pressurized to 3,500 pounds per square inch absolute (psia). The fixed volume of each railcar was 1,050 cubic feet. How many days did the helium in one railcar last for normal operations usage? *For extra help see resource links following Problem 2.*

**Problem 2**

When a Shuttle is launched approximately 1,000,000 standard cubic feet (scf) of helium are used to purge the Shuttle hydrogen systems, external tank, and main engines. How many railcars of helium are needed for each Space Shuttle launch? *For extra help see resource links below.*

***Resource Links for Problems 1 and 2***

Mathematical Aside: Direct and Inverse Proportions - *To model these gas problems we must understand the concepts of direct and inverse proportions. Gases obey several laws, each of which is either a direct or inverse proportion. If you want to learn more about direct and inverse proportions, click on the following link.*
Mathematical Aside: Direct and Inverse Proportions

Mathematical Aside: The Gas Laws - *Since helium is used by KSC in its gaseous form, we need to know several physical laws called the Gas Laws to be able to solve our problems. If you are not familiar with the Gas Laws, click on the following link.* Mathematical Aside: The Gas Laws

Glossary of Terms – *Terminology that might be helpful for your solutions to Problems 1 and 2 can be found at the following link.* Glossary of Terms

Hints: Problems 1 and 2

**Problem 3**

In the spring of 1998 KSC was required by law to use private vendors to acquire helium. The private vendors did not have helium railcars to supply the helium in gaseous form. Over-the-road gaseous helium tankers have a much smaller capacity than railcars, so it was not economically feasible to deliver gaseous helium using these containers. Liquid helium (LHe) is much denser than gaseous helium and can be carried more economically. The liquid helium can be converted to gas as needed through a vaporization process at KSC. To meet its helium needs, KSC now receives over-the-road liquid helium tankers. Liquid helium is sold by the liter, but KSC expresses its need in standard cubic feet (scf) of gas. The pressure and temperature for liquid helium in these tankers is approximately 47 psia and – 271.5° C, respectively. How much liquid helium in liters will KSC need on a daily basis for normal operations?

***Resource Links for Problem 3***

Mathematical Aside: Unit Analysis - *Because the given units of measurement are different from the units needed, we must use unit analysis to convert to the desired units. If you are not familiar with using unit analysis to convert between various units, click on the following link.*
Mathematical Aside: Unit Analysis

Glossary of Terms - *Terminology, including conversion factors, that might be helpful for your solutions to Problem 3 can be found on the following link.* Glossary of Terms

Hints: Problem 3

**Problem 4**

If a liquid helium tanker contains approximately 42,000 liters of liquid helium, how many days will the contents of one tanker last under normal operations?

**Problem 5**

Because of government regulations, KSC was required to switch to private vendors and to purchase helium in its liquid form. Previously KSC purchased gaseous helium at a cost of $0.07 to $0.08 per scf. Presently they are paying $2.00 per liter for the liquid helium. Has this required change been cost effective for KSC? Justify your answer.

***Resource Link for Problem 5***

Hints: Problem 5

## Mathematical Aside

## Direct and Inverse Proportions

### Direct Proportions

We say that an output variable y is directly proportional to an input variable x if y = kx where k is a constant. This implies that if x = 0 then y = 0, and that equal changes in x produce equal changes in y. Also, an increase in the magnitude of the variable x results in an increase in the magnitude of the variable y.

The following table gives data about the temperature (T) and the volume (V) of a confined gas.

**Table 1: Volume vs Temperature**

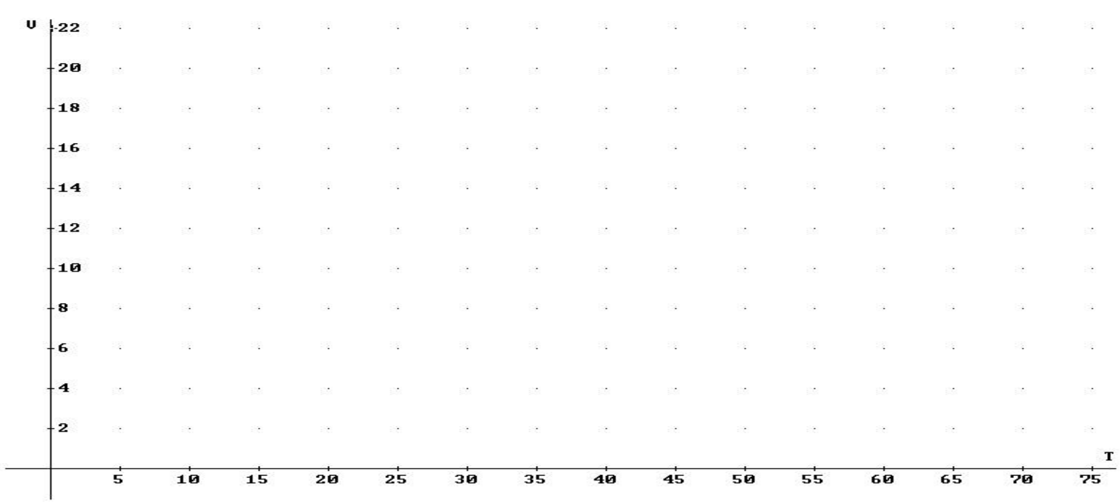
| Temperature (° C) | Volume ($in^3$) | Constant (k) |
|---|---|---|
| 25 | 10 | ? |
| 30 | 12 | ? |
| 40 | 16 | ? |

Do you see any patterns to the table? If you don't, try dividing each volume by its respective temperature. Place your answers in the "constant" column. Each division gives the same constant k, and this fact is characteristic of a direct proportion.

### Question

1) What is the value of the constant k for the direct proportion in Table 1?

Let's look at this direct proportion graphically. Using temperature for the horizontal axis and volume for the vertical axis, plot the data in Table 1 on the following coordinate system.




Connect the points to create a graph.

**Question**

2) What is the shape of the graph that passes through these points?

The straight line you drew through the data points is also characteristic of a direct proportion. Notice that as you extend your line to the left, it will pass through the origin (or should be close, depending on the accuracy of your points). You can always recognize a direct proportion graphically because it is a linear graph that passes through the origin. Equations associated with a graph like this can be written using the two equivalent forms below.

$$y = kx \quad \text{or} \quad \frac{y_1}{x_1} = \frac{y_2}{x_2}$$

**Question**

3) What is the linear equation that relates temperature and volume in Table 1?

**Inverse Proportions**

We say that an output variable y is inversely proportional to an input variable x if y = k/x where k is a constant. This implies that increases in the magnitude of the variable x result in decreases in the magnitude of the variable y. The following table shows the relationship between the volume of a gas and the pressure being applied to it. When mass and temperature are held constant, volume and pressure are inversely proportional.

**Table 2: Pressure vs Volume**

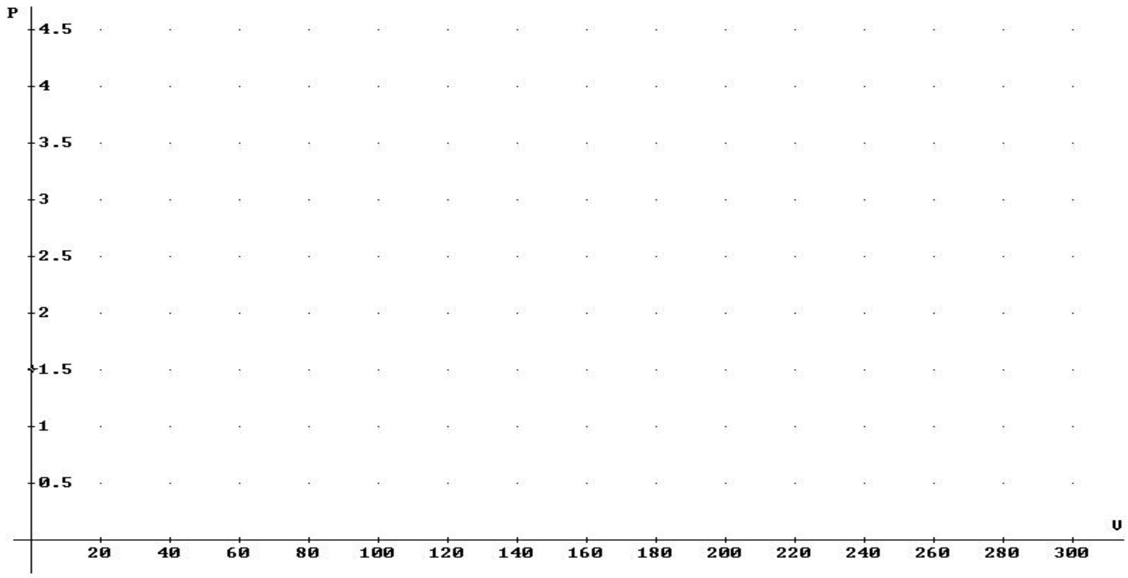
| Volume ($ft^3$) | Pressure (psi) | Constant (k) |
|---|---|---|
| 40 | 3.5 | ? |
| 50 | 2.8 | ? |
| 70 | 2 | ? |
| 140 | 1 | ? |
| 280 | 0.5 | ? |

Do you notice any patterns in Table 2? Recall the previous example where a constant was calculated from the quotient of temperature and volume. For an inverse proportion, we can determine a constant by multiplying the variables in each row.

**Question**

4) What is the value of the constant k for the inverse proportion in Table 2?

Let's examine the graph of an inverse proportion. Using volume for the horizontal axis and pressure for the vertical axis, plot the data in Table 2 on the following coordinate system.




Draw a smooth curve to connect the points. The graph relating pressure to volume has a very different shape compared to the linear graph of volume and temperature from Table 1. Notice the graph for Table 2 shows that pressure becomes very large as the volume approaches zero.

**Question**

5) As shown by the graph, what happens to pressure when the volume becomes very large?

Since the product of the variables is a constant, the equation is commonly written using one of the following three forms:

$$xy = k \quad \text{or} \quad y = k/x \quad \text{or} \quad \frac{x_1}{x_2} = \frac{y_2}{y_1}$$

**Question**

6) What is the equation that relates volume and pressure in Table 2?

**Exercises**

For the sets of data in Exercises 1 and 2 below, a) determine if the relationship is a direct or inverse proportion, b) determine the constant present in the relationship, and c) find the equation in the form $y = kx$ or $y = k/x$.

1)

| Temperature ( K) | Pressure (psi) | constant |
|---|---|---|
| 10 | 2000 | ? |
| 15 | 3000 | ? |
| 25 | 5000 | ? |

a) type of relationship:

b) constant:

c) equation:

2)

| Temperature (K) | Mass (kg) | constant |
|---|---|---|
| 10 | 20 | ? |
| 15 | 13.3 | ? |
| 25 | 8 | ? |

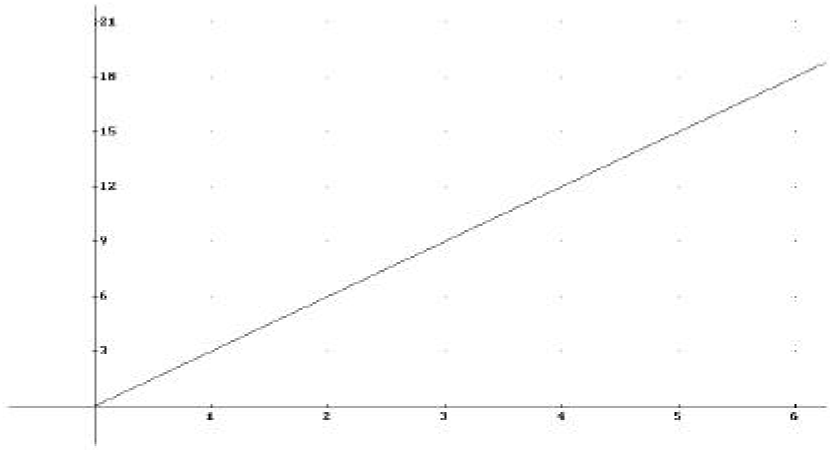
a) type of relationship:

b) constant:

c) equation:

For the graphs in Exercises 3 and 4, a) determine if the relationship is a direct or an inverse proportion, b) determine the constant present in the relationship, and c) find the equation in the form $y = kx$ or $y = k/x$ .

3)

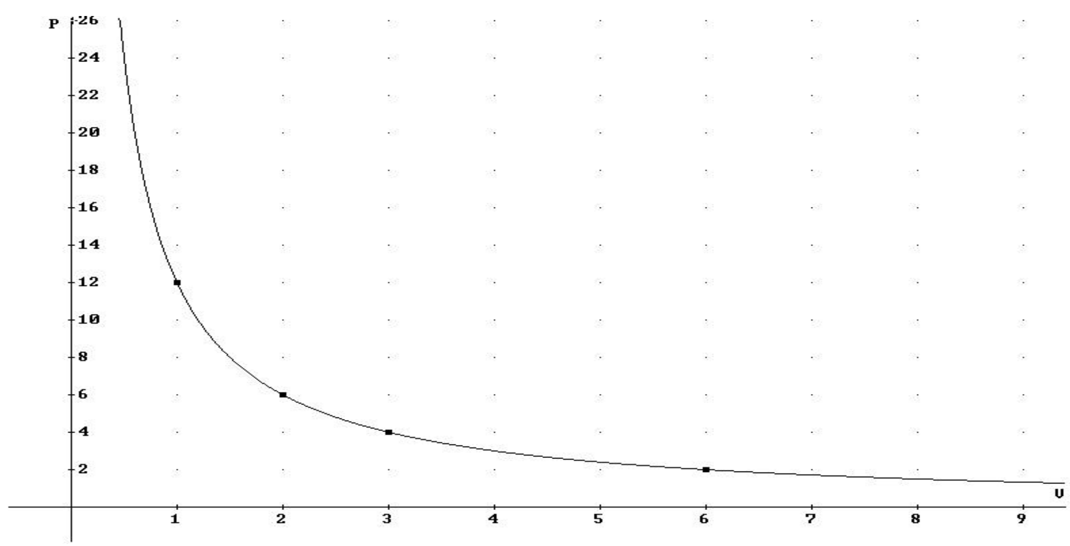



a) type of relationship:

b) constant:

c) equation:

4)




a) type of relationship:

b) constant:

c) equation:

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## Mathematical Aside

## The Gas Laws

### Charles' Law

Charles' Law describes how the volume and temperature of a gas are related if mass and pressure are kept constant. The table below shows several values of temperature and corresponding volume for a fixed mass of gas maintained at a constant pressure. The temperature values are expressed in the Kelvin scale, a scale based on the Celsius scale with – 273° C (absolute zero) set equal to 0 Kelvin (0 K). Notice that the degree symbol "°" used for Celsius and Fahrenheit temperature scales is not used for the Kelvin scale. We can convert between the Kelvin and Celsius scales using the formula K = C + 273. We must convert temperatures from Fahrenheit or Celsius to the Kelvin scale when using the Gas Laws.

| Temperature $T$ ( K) | Volume $V$ ($ft^3$) | constant $k$ |
|---|---|---|
| 240 | 40.0 | |
| 252 | 42.0 | |
| 255 | 42.5 | |
| 270 | 45.0 | |
| 276 | 46.0 | |
| 279 | 46.5 | |
| 291 | 48.5 | |

We can use a graphing calculator to graph this data to see if it is a direct proportion, an inverse proportion, or neither. One way of doing this is to create a scatterplot. To do this on the TI-83™, use these keystrokes.

STAT 4 {ClrList} 2nd L1 , 2nd L2 ENTER
STAT 1 {Edit}
Enter the temperature data in L1, arrow over to L2 and enter the volume data.
2nd STAT PLOT ENTER ENTER
Highlight the first type of graph icon {a scatterplot graph} ENTER
Xlist should be L1 and Ylist should be L2. If not, change them.
Make sure your window parameters match the data.
GRAPH

### Questions

1) Examine the graph of your data. From the graph, does it appear that Charles' Law is a direct proportion, an inverse proportion or neither? Justify your answer.

2) What is the value of the constant k for this proportional relationship?

Charles' Law states that the volume of a gas is directly proportional to the Kelvin temperature of the gas. This direct proportion can be expressed in either of the two following ways:

CHARLES' LAW

$$V = kT \quad \text{or} \quad \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

**Example 1**

A 30 liter container holds 1 mole (a measure of mass) of oxygen gas at 20° C. If the temperature is raised to 30° C while the mass and pressure remain constant, what volume container is needed to hold the oxygen gas?

Solution

Since the mass and pressure remain constant, we can use Charles' Law. We first change the temperatures to the Kelvin scale:

$$K = C + 273 \Rightarrow 20° \text{ C} = 293 \text{ K} \quad \text{and} \quad 30° \text{ C} = 303 \text{ K}$$

We can now substitute into Charles' Law as follows:

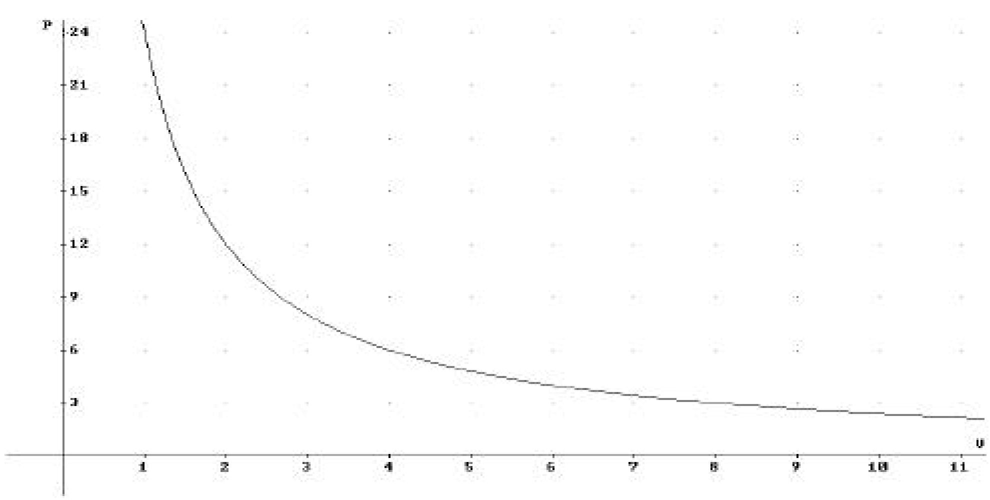
$$\frac{30 \text{ liters}}{293 \text{ K}} = \frac{V_2}{303 \text{ K}}$$

$$V_2 = \frac{30(303)}{293} = 31$$

Thus, $V_2 = 31$ liters.

**Boyle's Law**

The graph below is an example of the relationship between the volume of a gas and the pressure exerted by the gas when the temperature and mass are held constant.



**Question**

3) Based on the shape of the graph, is this relationship a direct proportion, inverse proportion or neither? Justify your answer.

**Boyle's Law**

Boyle's Law states that the pressure exerted by a gas is inversely proportional to its volume. This inverse relationship can be expressed in either of the two following ways.

BOYLE'S LAW

$$P = \frac{k}{V} \quad \text{or} \quad P_1V_1 = P_2V_2$$

**Example 2**

A gas has a volume of 400 ml at a pressure of 1000 psia. What will the volume be if the pressure is increased to 2000 psia when the temperature and mass are held constant?

Solution

$$P_1V_1 = P_2V_2$$
$$1000(400) = 2000(V2)$$
$$200 \text{ ml} = V_2$$

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## Glossary of Terms

**Absolute zero** is the temperature at which the atoms in a substance stop moving.

**Atmospheric pressure** is the pressure on the surface of the earth due to the atmosphere, the envelope of air around the earth. One atmosphere (abbreviated atm) is approximately 14.7 pounds per square inch.

**Boyle's Law** is the gas law that states that the volume of a gas of fixed mass and temperature is inversely proportional to the pressure applied to the gas.

**Celsius** is a temperature scale set so that the freezing point of water at standard pressure (one atmosphere) is 0° C (Celsius) and the boiling point of water at standard pressure is 100° C. The Celsius is also called the centigrade scale. The formula K = C + 273 can be used to convert between Celsius and Kelvin scales.

**Charles' Law** is the gas law that states that the volume of a gas with a fixed pressure applied to it and a fixed mass is directly proportional to the temperature on the Kelvin scale.

The **Combined Gas Law** is a combination of several gas laws and states that if the mass of a gas is constant, then the volume is inversely proportional to the pressure and directly proportional to the temperature on the Kelvin scale.

A **direct proportion** is a relationship between two variables in which the ratio of the associated values of the two variables is a constant.

**Fahrenheit** is the common temperature scale commonly used in the United States. On this scale the freezing point of water at standard pressure is 32° F, and the boiling point of water at standard pressure is 212° F. The formula C = 5/9(F – 32) can be used to convert between Fahrenheit and Celsius scales.

A **gas** is an easily compressible fluid. A given mass of gas assumes the shape and size of its container.

An **inverse proportion** is a relationship between two variables in which the product of the associated values of the two variables is a constant.

**Kelvin** is the *System Internationale* (SI) base unit of temperature in which absolute zero (the theoretically lowest possible temperature) is 0 K (Kelvin). The formula C = K – 273 can be used to convert between the Kelvin scale and Celsius scales.

A **liter** (*l*) is a measure of volume in the SI metric system. One liter is one cubic decimeter ($dm^3$).

**Pressure** is a measure of the number of collisions between particles and of collisions between particles and the walls of a container. Pressure is the force exerted per unit area of a surface.

A **proportion** is a statement that two ratios are equal.

**Standard cubic feet** (scf) is a unit of volume used in the purchase and use of industrial gases. One scf is 1 cubic foot of gas at 70° F and 14.7 psia (1 atm). One cubic foot equals 0.0283 cubic meters. $1\ ft^3 = 0.0283\ m^3$

**Standard temperature and pressure** (STP) is 0° C and 1 atm.

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## Hints for Problems 1 and 2

### 1) Understand the problem.

In order to know which gas law to use, we need to know which of the potential variables of pressure, volume, mass, and temperature remain constant in this situation. To make use of the helium in a railcar, it is released into pipes which lead to various locations where the helium is used. The pressure in the pipes is 1 atmosphere and the temperature is the ambient temperature (approximately 70° F).

Does the pressure change when the helium is released from the railcar? Explain your reasoning.

Does the mass (the number of helium molecules) change when the helium is released from the railcar? Explain your reasoning.

Does the temperature change when the helium is released from the railcar? Explain your reasoning.

### 2) Devise a plan.

Based on your answers to the above questions, which gas law (Boyle's or Charles') should be used in solving this problem? How will the use of this law help you to solve this problem?

### 3) Carry out the plan.

How many standard cubic feet (scf) of gaseous helium could be obtained from one railcar filled with liquid helium? How many days will this amount of gaseous helium last under normal operations?

### 4) Look back.

What did you learn from solving this problem?

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## Mathematical Aside

## Unit Analysis

Unit analysis is used to convert from one unit of measurement to a different unit of measurement. The conversion process is done by multiplying the original value by unit fractions set up to produce the new units of measurement. Unit fractions are fractions that equal the number one. For example, you might have a measurement of 36 inches but want to know this length in feet. For this conversion, we know that 12 inches equals 1 foot. This fact gives two unit fractions:

$$\frac{12 \text{ inches}}{1 \text{ foot}} \quad \text{or} \quad \frac{1 \text{ foot}}{12 \text{ inches}}$$

To decide which of these unit fractions to use, consider the following multiplications:

$$36 \text{ inches} \cdot \frac{12 \text{ inches}}{1 \text{ foot}} = \frac{36 \text{ inches} \cdot 12 \text{ inches}}{1 \text{ foot}}$$

$$36 \text{ inches} \cdot \frac{1 \text{ foot}}{12 \text{ inches}} = \frac{36 \text{ inches} \cdot 1 \text{ foot}}{12 \text{ inches}} = 3 \text{ feet}$$

In the second multiplication, the “inches” label divides out (or cancels), leaving the desired label “feet”.

**Example 1**

Convert 4800 seconds into hours.

Solution

We know that 60 seconds = 1 minute, and that 60 minutes = 1 hour. Thus,

$$\frac{4800\text{sec}}{1} \cdot \frac{1\text{min}}{60\text{sec}} \cdot \frac{1\text{hour}}{60\text{min}} = \frac{4}{3} \text{hours}$$

or 1 hour and 20 minutes (Why?)

Notice that:

- all of the units divided out ( or cancelled) except for the unit “hours” which was the label we wanted from the conversion, and
- all of the conversion fractions have a value of “1” since the numerator and denominator are equal to one another.

**Example 2**

This year we want to replace the indoor/outdoor carpeting on an outdoor patio floor with terazzo tile, which is measured in square centimeters. We used 12 square yards of carpeting when we put the carpet in several years ago. To replace this carpet with tile, we need to know the area of the patio in square centimeters.

Solution

In order to do a unit analysis, we need to know the conversion factors between the units we begin with and the units we want (as in 60 minutes = 1 hour or 12 inches = 1 foot). These factors can be found in various places, such as dictionaries and other reference books. For Example 2, we will use the following: 1 foot = 0.03048 meters, 3 feet = 1 yard, and 100 centimeters = 1 meter. Since our problem involves area, we will use square units. Thus,

1 square foot = 0.0929 square meters
1 square yard = 9 square feet
10,000 square centimeters = 1 square meter

We will use these conversions to change 12 square yards to an equivalent area measured in square centimeters. Remember to set up the unit fractions so that the appropriate labels will divide out (or cancel).

$$\frac{12\text{sq yd}}{1}\cdot\frac{9\text{sq ft}}{1\text{sq yd}}\cdot\frac{0.0929\text{sqm}}{1\text{sqft}}\cdot\frac{10{,}000\text{sqcm}}{1\text{sqm}}=100{,}332\text{sqcm}$$

**Exercises**

1) A sprinter runs the 100 m dash in 10 seconds. Assume this speed can be maintained.
   a) What is the equivalent speed in miles per hour? (1 m = 3.28 ft)?
   b) How long would it take to run a marathon that is 26 miles 385 yards?

2) An astronaut experiences pressure of 3 atm.
   a) Express this pressure in psia.
   b) Express this pressure in kiloPascals (kPA). (1 psi = 6.89 kPA)

3) The surface of the Orbiter Endeavor requires approximately 27,000 tiles that each cover an average surface area of 500 sq cm. Approximate the surface area of this Shuttle in square feet.

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## Hints for Problem 3

### 1) Understand the problem.

In order to know which gas law to use we need to know which of the potential variables of pressure, volume, mass, and temperature remain constant in this situation. To use the helium in a railcar, it is released into pipes that lead to the various locations that use helium. The pressure in the pipes is 1 atmosphere and the temperature is the ambient temperature (approximately 70° F).

Does the pressure change when the helium is released from the railcar? Explain your reasoning.

Does the mass (the number of helium molecules) change when the helium is released from the railcar? Explain your reasoning.

Does the temperature change when the helium is released from the railcar? Explain your reasoning.

### 2) Devise a plan.

Based on your answers to the above questions, which gas law (Boyle's, Charles' or the Combined Law) should be used in solving this problem?

How will the use of this law help you to solve the problem?

In order to use a gas law involving temperature, all temperatures must be in the Kelvin scale. Convert the temperatures in this problem to the Kelvin scale.

### 3) Carry out the plan.

In order to determine how much liquid helium (LHe) Kennedy Space Center will need on a daily basis, 70,000 scf must be converted to liters. (1 liter = 1 cubic decimeter)

Use unit analysis to convert 70,000 cubic feet to cubic meters.

Convert your answer to cubic decimeters. Use the appropriate gas law to determine the number of liters of LHe that Kennedy Space Center uses on a daily basis for normal operations.

### 4) Look back.

How did you use proportions in solving this problem? Could this problem have been solved in a different way? Explain.

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## Hint for Problem 5

How much did Kennedy Space Center pay for 70,000 scf of gaseous helium at $0.075 per scf?

How much does Kennedy Space Center pay for the equivalent number of liters of LHe at $2.00 per liter?

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