LTA 1

NASA - AMATYC - NSF
Project Coalition

Kennedy Space Center

Spare Parts for the Space Shuttle

Mathematics for Engineering Technology

Industrial and Management Manufacturing

Capital Community-Technical College

STS-84 Crew Members, from left, Mission Specialist Carlos I. Noriega,
Commander Charles J. Precourt, and Mission Specialist Jean- Francois Clervoy examine
the tires of the Space Shuttle Atlantis after landing.

LTA 1

Spare Parts for the Space Shuttle

Mathematics for
Industrial and Management Engineering Technology
Manufacturing Engineering Technology

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LTA 1

Spare Parts for the Space Shuttle

How much did that last set of tires for your car cost you? What made you decide that you needed to buy them – a newspaper ad, a flat tire? How are you going to save all the work you've done on your English paper – on computer diskettes? Did you buy enough? How do you know? All of us have to make purchasing decisions all the time – how much to buy, when to buy it.

When a Shuttle is launched, the general public is focused on the astronauts in space and the engineers at the Johnson Space Center in Houston. However, few of us are aware of the many engineers and scientists who work for years on various aspects of the mission. One such group are the scientists and engineers who determine how many spare parts should be kept on hand to insure that a spare is available when needed. They determine the number of spares needed for each of the 250,000 repairable and non-repairable parts. After this determination is made, there are many other people involved in repairing, cataloging, and storing the parts to make them readily available at a moment's notice.

Seth Berkowitz, Randy Greeson, and Marcia Groh-Hammond are logistics engineers at Kennedy Space Center. Here are excerpts of what they have to say about their job:

> There are certain things in life that we take for granted. For instance, we never give much thought to it, but we keep spares of basic household products. Most households contain spare paper towels, toilet paper, soap, shampoo, etc. How many and what types of items we buy to support our house is usually automatically calculated in our heads while going through the supermarket. We consider how often we consume an item, and how much it costs. Generally, the more expensive the item is the fewer spares we keep on hand. We are concerned about our budget. We know in most cases we can replenish that item when we need it; it is simply a matter of going to the store to pick it up. For instance, you are making a cake and run out of eggs. You go to the store and buy what you need. There is no waiting for the hen to lay the eggs and the eggs to be delivered from the farm. It's a short delay, a slight inconvenience.
>
> Now imagine not being able to go to the corner grocery store and pick up what you need. On top of that, you have to wait weeks, months, maybe longer, once you decide you need it. The items you are buying cost $1,000, $10,000, $100,000 or more than $1,000,000. Deciding what you need and when to buy it is not as automatic as buying ordinary household items. This is the dilemma that faces the Space Shuttle Logistics Office.
>
> One of the keys to successfully launching Shuttles on-time and within budget involves hardware availability. The availability of the hardware must not impose a constraint on the processing of the Shuttles. The Nation cannot afford to have an expensive national resource such as the Shuttle waiting on the launch pad for a part. The methods used to determine spare requirements for the Shuttle must be effective and efficient in terms of adequately identifying the required spares without incurring excessive program costs.

This LTA (Laboratory Technical Activity) will give you an opportunity to determine appropriate inventory levels for fuel cells, a repairable item, and for main landing gear tires, a non-repairable part. In Section 1 you will find how many fuel cells must be kept on hand in order to be reasonably confident that one is available when needed. In Section 2 you will determine how many main landing gear tires to keep on hand (Minimum Stock Level) and how many to order (Order Quantity).

Section 1

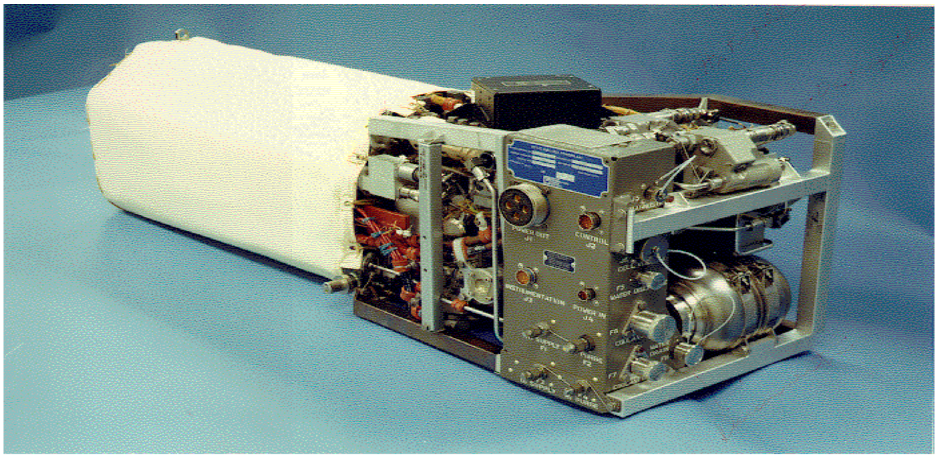
Line Replaceable Units

Fuel Cells

Many Shuttle parts can be repaired when they fail. These are referred to as repairable line replaceable units (LRUs). There are over 250,000 unique parts on a Shuttle. Nine thousand of these parts are considered repairable line replaceable units (LRUs). The following excerpts from what logistics engineers Seth Berkowitz, Randy Greeson, and Marcia Groh-Hammond said about LRUs will introduce you to some of the factors that go into deciding the inventory levels for repairable items.

> Failure of a repairable LRU requires removal of the unit and replacement with a spare unit. The failed unit is sent out for repair; repair time ranges from days to years depending on the degree of failure. This repair turnaround time or hardware inavailability time is one of the major factors that is considered when determining the number of spares needed for repairable hardware. Other factors are a unit's failure rate, the number of units per Shuttle, and the amount of time that the unit is used in-flight and on-the-ground. These parameters are based on actual historical data as well as analyses and projections. Engineering evaluation of all the data is performed before the value of a specific parameter is determined. For example, if a unit is modified to eliminate a specific failure mode, the failure rate for the unit would be different before and after the modification. Other examples include moving the unit repair location from the manufacturer to a repair depot; enhancing technician training to reduce human error handling mishaps; and upgrading equipment to provide for a more efficient repair process.
>
> All of the data plays a key role in determining the inventory levels. For the repairable items, a probability of sufficiency (POS) equation is used to determine the appropriate inventory level for spares. This POS equation is based on the Poisson probability distribution. The Shuttle program has a goal for repairable hardware of 90% POS, which means that nine out of ten times a spare unit will be available when needed.
>
> POS is calculated by logistics engineers (LEs) who do not necessarily take the results of the equation at face value. The POS equation is a tool and does not replace logistics engineers' experience and expertise. LEs often perform rigorous exercises or trade-off studies before determining the appropriate inventory level for an individual LRU. These exercises generally utilize the POS equation. The individual parameters are manipulated by the LEs. By doing this manipulation, the LEs can determine the costs and benefits of changing the inventory requirements. For example, will decreasing the repair turnaround time by 10% decrease the number of spares required? The cost of reducing the repair turnaround time (paying for twenty-four hour a day repair service or moving the repair location to an alternative facility) is compared to the cost of purchasing another unit.

One of the most important line replaceable units is the fuel cell (Figure 1). Each new fuel cell costs more than $7 million dollars. Can you see why it is necessary to limit the number of spare fuel cells? The cost to repair a fuel cell is $3 million dollars or more, still a lot of money but much less than the cost of a new one.



United Technologies Power Systems - Space Shuttle Orbiter Fuel Cell

Figure 1 – Fuel Cell

Fuel cells supply electricity to the Shuttle's avionic components. The cells generate electricity through a chemical reaction that mixes a fuel (hydrogen) with an oxidizer (oxygen). The reaction also produces water which is used as a vehicle coolant and for drinking by the crew. There are three fuel cells on each Shuttle, two on the right side and one on the left.

The fuel cell dimensions are 45" long, 15" wide, and 14" high. The fuel cell weighs 262 pounds dry and 281 pounds with coolant. The maximum power output, when installed in the Shuttle, is 16 kilowatts, and the voltage is regulated to stay in the range 27.5 to 32.5 volts dc.

In the following, consider that you are a KSC engineer with responsibility for determining how many spare fuel cells should be kept on hand. In order to make this determination, you must first evaluate two quantities that affect the inventory level. These are the Maintenance Demand Rate (MDR) and the Total Projected Operating Time (TPOT).

Part A Calculating the Maintenance Demand Rate (MDR)

The Maintenance Demand Rate (MDR) is the total number of failures of a part per 1000 operating hours experienced in the past.

$$\text{MDR} = \frac{\text{Total Failures}}{\text{Total Hours Experienced in Past}} * 1000$$

First of all, you search the failure history data base and find that the total number of fuel cell failures as of May 1997, is 46. Next, you determine the total number of hours that the fuel cells have accumulated. To do this, you must know how many hours the fuel cells are used on each flight, the number of fuel cells on each Shuttle, and the number of flights.

1) QPV stands for the Quantity Per Vehicle, the number of fuel cells on each Shuttle.
   How many fuel cells are on each vehicle?______________ (Refer to the description of the fuel cells given previously.)

The number of hours that the fuel cells are used on each flight is dependent upon the number of hours used in flight, or the Flight Power-On Time (FPOT) per flight, and the number of hours used while the Shuttle is still on the ground, called Ground Power-On Time (GPOT) per flight. For these fuel cells the Flight Power-On Time (FPOT) is 230 hours, and the Ground Power-On Time (GPOT) is 36 hours.

The number of hours that the fuel cells have already been used (Total Hours Experienced in Past) also depends on the number of Shuttle flights. In May 1997, NASA calculated this number by using the total number of flights, starting with the 31st mission, that have taken place since the Challenger accident. As of May 1997, the total number of flights was 53.

Total Hours Experienced in the Past = QPV*(FPOT+GPOT)*Total Flights

2) Calculate the Total Hours Experienced in the Past.
   a) Quantity Per Vehicle (QPV) = _________
   b) Flight Power-On Time per flight (FPOT) = _________
   c) Ground Power-On Time per flight (GPOT) = _________
   d) Total Flights = _________
   e) Total Hours Experienced in the Past = _________

3) Now use the formula given for the Maintenance Demand Rate and calculate the MDR:
   a) Total failures = _________
   b) Total Hours Experienced in Past = _________
   c) Maintenance Demand Rate (MDR) = _________
   d) State in words what this number represents: This means _____failures in ____hours.

Part B Calculating the Total Projected Operating Time (TPOT)

Your next step is to predict the number of hours that the fuel cells will be in use during the upcoming year. This is referred to as the Total Projected Operating Time (TPOT). As of May 1997, NASA calculates this using a flight rate of 8 flights per year.

1) Using the values for FPOT and GPOT from Part A, find the Total Projected Operating Time for fuel cells from the following formula. Remember the number of fuel cells on each Shuttle, the QPV, equals 3.

   Total Projected Operating Time (TPOT) = QPV*(FPOT+GPOT)*Flight Rate Per Year
   a) Quantity Per Vehicle (QPV) = _________
   b) Flight Power-On Time per flight (FPOT) = _________
   c) Ground Power-On Time per flight (GPOT) = _________
   d) Flight rate = _________
   e) Total Projected Operating Time (TPOT) = _________
   f) In your own words write a sentence describing what this number represents.

You are now ready to determine the probability that a spare fuel cell will be available when needed.

Part C Calculating the Probability of Sufficiency (POS)

The Probability of Sufficiency (POS) gives the engineer the probability that a spare part will be available when needed. For example, a Probability of Sufficiency (POS) of 0.75 indicates that there is a 75% chance of the part being ready to use if necessary. The Shuttle program has a minimum goal for repairable hardware of 90% Probability of Sufficiency which means that nine times out of ten a spare unit will be available when needed. The engineers use the following formulas which were developed using the Poisson probability distribution from statistics.

Before we can find the POS, we must calculate the probability of exactly n failures in a time period T. When a part fails, it must be replaced by a spare part from inventory, thus decreasing the number of spare parts available. This means that the number of parts removed from inventory (removals) is equal to the number of parts that fail (failures).

The probability of exactly n removals, or failures, in time T for a part with a removal or failure rate λ (lambda) per unit time equals

$$P(n) = \frac{(\lambda T)^n * e^{-\lambda T}}{n!}$$

where

n = the number of removals (or failures),
T = repair turnaround time of the item in days, and
λ = removal (or failure) rate per day.

For a time T, n is the number of units that have been removed due to failure and need repair (and so are not available for use during the turnaround time). For example, suppose that n = 4, T = 200 days, and P(4) = 0.13. This means we have a 13% chance that 4 units will fail during time T, the 200 day turnaround time to get the unit repaired. This is equivalent to saying that there is a 13% chance that 4 units will be removed from inventory during the 200 day turnaround time.

Now the engineer can calculate the Probability of Sufficiency (POS) by finding the sum, $\sum$, of the probabilities of exactly n removals during time T for n = 0, 1, 2, … up to the total number of spares, S. The Probability of Sufficiency of a part with S spares is:

$$POS = \sum_{n=0}^{S} P(n) = \sum_{n=0}^{S} \frac{(\lambda T)^n * e^{-\lambda T}}{n!}$$

POS is the probability of having a spare of a particular item available when required. To determine the Probability of Sufficiency, you first must calculate lambda (λ) the expected removal rate per day. The formula for λ is:

$$\lambda = \frac{MDR}{1000} * \frac{TPOT}{365}$$

1) Answer the following questions with reference to the formula for λ.
   a) What does the 1000 represent? (Refer to the section on the Maintenance Demand Rate, or MDR.)
   b) What does the 365 represent?

2) Calculate λ by filling in the following blanks.
   a) Maintenance Demand Rate MDR = __________
   b) Total Projected Operating Time TPOT = __________
   c) Expected removal rate per day λ = __________

We can now use this value of λ to determine the probability of exactly n failures, P(n), for various values of n. These values can then be summed to obtain the POS. Recall that the formula for P(n) is:

$$P(n) = \frac{(\lambda T)^n * e^{-\lambda T}}{n!}$$

3) Let's begin by looking at P(0), the probability that there are no fuel cell failures for a turnaround time of T = 250 days.
   a) In this case, n = _________ T = _________ and λ = __________

   b) $P(0)$ = ___________________

Your calculations should show a probability of almost 1%. In other words, there is a very small probability that there will be no fuel cell failures during the 250-day time period. This means that there is a very high probability that there will be fuel cell failures resulting in removals from inventory.

4) Consider the possibility of exactly one fuel cell removal in a 250-day turnaround time.
   a) n = _________ T =_________ and λ = __________

   b) $P(1)$ = ____________

5) Using this same process, calculate the value of P(2).

   $P(2)$ = ____________

6. a) Now find the Probability of Sufficiency (POS) for S = 2.

$$= P(0) + P(1) + P(2)$$

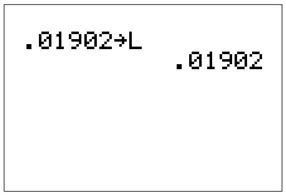
$$POS = \sum_{n=0}^{2} P(n) = \sum_{n=0}^{2} \frac{(\lambda T)^n * e^{-\lambda T}}{n!}$$

= ________ + ________ + ________ = __________

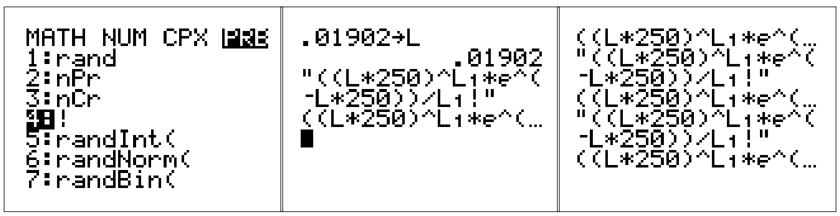
   b) Write a sentence explaining what this probability means.

Part D Calculating the Probability of Sufficiency (POS) Using a TI-83™ Graphing Calculator

1) Calculate Lambda and store in L.



2) Hint: To make the entry of a long formula into each list easier you can cut and paste it from the Home Screen. Before entering formulas into the actual lists go to the Home Screen and enter the formula "((L*250)^L1*e^(-L*250))/L1!" and press ENTER. The ! is located under MATH,PRB, #4. Repeat this four times by pressing 2nd ENTER, then ENTER four times.

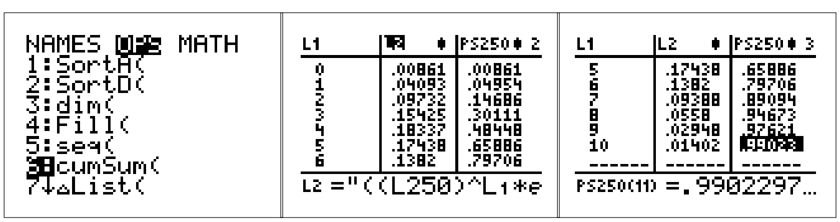


You will be able to cut and paste this formula into the lists you use to calculate *P(n)*, and then edit the formula for the various turnaround times.

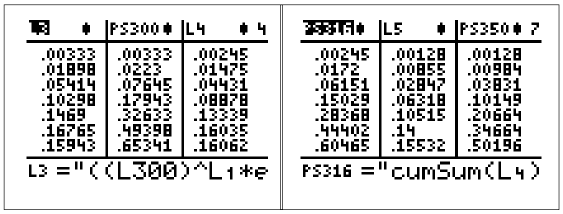
Enter the numbers 0-10 in L1 to represent the number of spares.

For T = 250, calculate *P*(*n*) in L2 by highlighting L2, pressing 2nd ENTER to cut and paste the formula from the Home Screen, and pressing ENTER. If you put the formula in quotation marks you will be attaching it to the list. The list will have spreadsheet capabilities and remember this formula for future calculations.

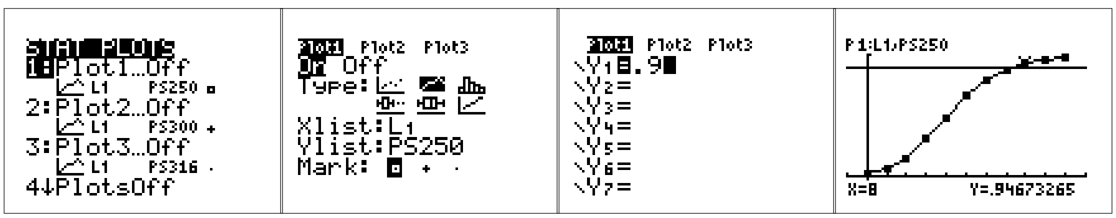
To calculate the POS for T = 250, name a list by highlighting L3, pressing 2nd Insert, and typing PS250. Notice that when you name a list the cursor is in alpha mode. Press alpha to enter the numbers. In list PS250 attach the formula PS250 = "cumSum(L2)" which is the POS for a 250 day turnaround time. CumSum is located under 2nd STAT,OPS, #6. Press ENTER.



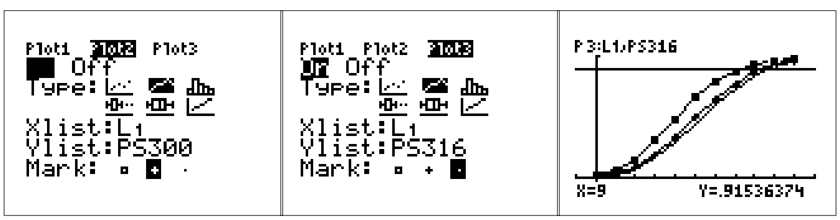
3) Do the same procedure for L3, PS300, L4, PS316, and L5, PS350 using the following entries.
L3 ="((L*300)^L1*e^(-L*300))/L1!" -- highlight L3, press 2nd ENTER, edit 250 to 300,
PS300="cumSum (L3)"
L4 ="((L*316)^L1*e^(-L*316))/L1!" -- again use 2nd ENTER and edit feature
PS316="cumSum (L4)"
L5 ="((L*350)^L1*e^(-L*350))/L1!" -- again use 2nd ENTER and edit feature
PS350 ="cumSum (L5)"



4) Graphing the POS:
To view a graph of the POS, go to STAT PLOT and turn on L1 and PS250. (To cut and paste the name of List PS250, press 2nd List and find the corresponding name to paste.) In Y=, graph Y1 = 0.90 to see which plots are above the 90% level. To get an appropriate window, press Zoom 9 which is ZoomStat.



5) Since you can only graph three stat plots at a time, choose to graph for T = 250, 300, and 316. Your graph should look like the following graph.



6) Now fill in the following table for the POS. There is a worksheet at the end of Section 1 to help you with these calculations. Remember that the POS indicates the probability of a spare being there if a failure occurs. Calculate the POS for:

T = 250, 300, 316, and 350 days
S = 0 to 10 total spares

Table 1

S (Total spares)	0	1	2	3	4	5	6	7	8	9	10
POS (T = 250)											
POS (T = 300)											
POS (T = 316)											
POS (T = 350)											

7. a) Engineers working on the Shuttle program have a goal of 90% Probability of Sufficiency (POS). To achieve the goal of 90% how many spares would be needed for a turnaround time of 250 days? 300 days? 316 days? 350 days?

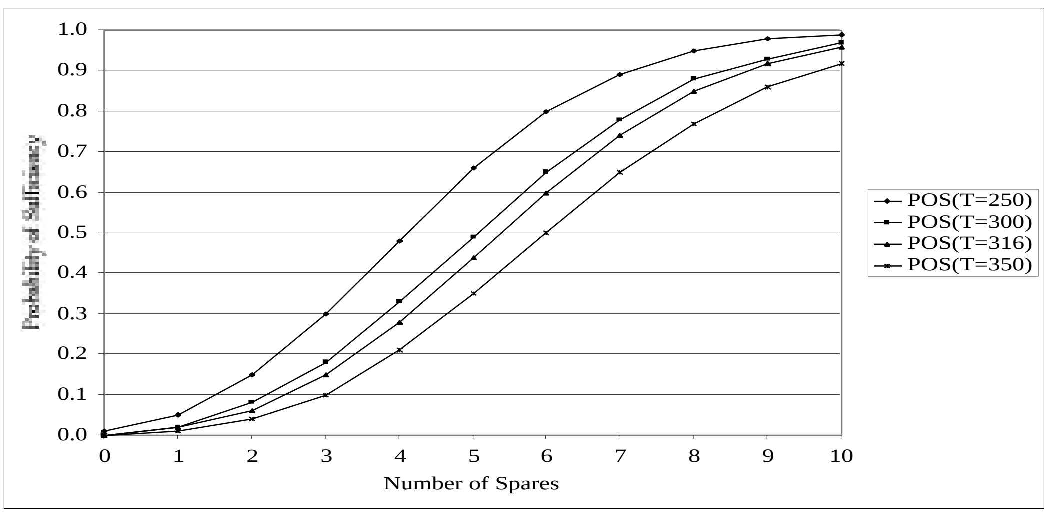
   b) What is the relationship between the number of spares required to attain a 90% POS and the turnaround time? Verify your answers with graphs in Exercise 5 as well as with the table in Exercise 6.

   c) Complete the sentence: If the turnaround time (T) could be decreased, the number of required spares would ______________________.

   d) Of the given possible turnaround times (T), NASA would like T to be _____. Why?

In 1997 the actual value of the turnaround time (T) used by the NASA engineers in determining the number of spare fuel cells needed, based on data from previous repairs, was 316 days. The following Excel™ graph shows the POS for the values of S from 0 to 10 and T = 250, 300, 316, and 350.

Figure 1



8) You see 4 different graphs, one for each turnaround time. What effect does increasing the turnaround time have on the Probability of Sufficiency for 5 spares? Does the probability of having a spare available increase or decrease with longer turnaround times?

9) For a 250 day turnaround time, as you increase the number of spares what happens to the Probability of Sufficiency? What does this mean in practical terms?

Worksheet: **POS Calculation for Fuel Cells**

Total Hours Experienced in the Past = QPV * (FPOT + GPOT) * Total Flights

QPV =
FPOT =
GPOT =
Total Flights =
Total Hours =

MDR = (Total Failures/Total Hours) * 1000

Total Failures =
Total Hours =
MDR =

Total Projected Operating Time = QPV * (FPOT + GPOT) * Flight Rate

QPV =
FPOT =
GPOT =
Flight Rate =
TPOT =

Lambda = (MDR/1000)*(TPOT/365)

Lambda =

T (Repair turnaround time) = 250 300 316 350

S (Total Spares)	0	1	2	3	4	5	6	7	8	9	10
POS(T=250)											
POS(T=300)											
POS(T=316)											
POS(T=350)											

Section 2

Non-repairable Parts

Main Landing Gear Tires

Many Shuttle parts are not repairable. They must be replaced with new parts when they wear out. For instance, a Shuttle has two types of tires: two nose landing gear tires in front and four larger main landing gear tires behind them. When the Shuttle lands, the four main landing gear tires hit the ground first, followed by the two nose landing gear tires. For this reason the nose landing gear tires at the front of the Shuttle receive less wear and can be used twice before being replaced, while the main landing gear tires are replaced after each flight.

It is of course necessary to have non-repairable items on hand when they are needed. How do you make sure that you have a replacement when a bulb burns out in your house? You probably keep a few new bulbs on hand and purchase more when very few are left. The Logistics Engineers do exactly this for each non-repairable item. They determine how many of each item to keep in stock, and how many to order. These are referred to as the Minimum Stock Level and the Order Quantity respectively.

Main landing gear tires are non-repairable items with a life of one landing. Each Shuttle has four main landing gear tires in two dual-wheel configurations. The tires are inflated with gaseous nitrogen to a pressure of 315 psi. The maximum allowable load per main landing gear tire is 123,000 pounds, and they are rated at 258 miles per hour. The tires cost $5,560 in May 1997 and are made especially for the Shuttles.

You now turn to the problem of determining how many main landing gear tires to keep in stock and how many to order so that a sufficient, but not excessive, number of new tires is available when needed.

Part A Calculating the Minimum Stock Level (MSL)

The Minimum Stock Level (MSL) is the number of items that should be kept in stock. When the number of items in stock declines to the MSL, it is time to order more of those items. The Minimum Stock Level is determined by looking at the number of items needed for each flight (the "issue rate"), the number of flights per year, the amount of lead time needed for the order (to account for things like delivery time), and a built-in "safety pad" of 12 months of stock. The "safety pad" accounts for the possibility of a problem with the order or a higher demand for the item than had been anticipated.

$$\text{MSL} = \left(\frac{\text{average issue rate}}{\text{flight}} \times \frac{\text{flights}}{\text{year}} \times \frac{1 \text{ year}}{12 \text{ months}}\right) \times (\text{lead time} + \text{safety pad of 12 months})$$

1) What effect does the "1 year / 12 months" have on the equation?

2) For the main landing gear tires, what is the issue rate (that is, how many main landing gear tires are used on each flight)?_________

3) As of May 1997, the number of flights per year is 8 and the lead time for ordering the main landing gear tires is 9 months. Fill in the following blanks and calculate the Minimum Stock Level.

$$\text{MSL} = \left(\frac{\_\_\_\_\_\ \text{tires}}{\text{flight}} \times \frac{\_\_\_\_\_\ \text{flights}}{\text{year}} \times \frac{1\ \text{year}}{12\ \text{months}}\right) \times \left(\_\_\_\_\_\ \text{months} + 12\ \text{months}\right)$$

$$= \_\_\_\_\_\_\_\ \text{tires}$$

4) Write a sentence to explain what your answer means.

Part B Calculating the Order Quantity (OQ)

The Logistics Engineers use the Minimum Stock Level to decide whether to order more parts. If the number of items on hand (the number of serviceable parts, or "Ser.") is more than the MSL, then no items need to be ordered. However, if the number of serviceable parts is at or below the MSL, the engineers must order more parts. The Order Quantity (OQ) tells the Logistics Engineers how many parts to order. It depends on the issue rate, the number of flights per year, the support period required (the amount of time the quantity of items ordered is expected to last, in this case 12 months), the number of items already ordered (the "Due-In Quantity"), and the number of items already on hand. Keep in mind that items are ordered when the number of serviceable parts on hand (Ser.) is less than or equal to the MSL. If Ser. exceeds the MSL, there is a sufficient number of items in stock and the order quantity is zero (no order required).

$$\text{OQ} = \left[\left(\frac{\text{average issue rate}}{\text{flight}} \times \frac{\text{flights}}{\text{year}} \times \frac{1\ \text{year}}{12\ \text{months}}\right) \times \left(\text{support period required}\right)\right] + \text{MSL} - \text{Due-In Qty.} - \text{Ser.}, \quad \text{if MSL} \geq \text{Ser.}$$

$$\text{OQ} = 0, \quad \text{if MSL} < \text{Ser.}$$

1) For the main landing gear tires
   a) Average issue rate / flight = ___________
   b) Flights / year = ___________
   c) Support period required = ___________
   d) Minimum Stock Level (MSL) = ___________
   e) Due-In Qty. = 0 tires
   f) Ser. = 54 tires
   g)

$$\text{OQ} = \left[\left(\frac{\_\_\_\ \text{tires}}{\text{flight}} \times \frac{\_\_\_\ \text{flights}}{\text{year}} \times \frac{1\ \text{year}}{12\ \text{months}}\right) \times 12\ \text{months}\right] + \_\_\_\ \text{tires} - \_\_\_\ \text{tires} - \_\_\ \text{tires}$$

$$= \_\_\_\_\_\_\_\ \text{tires}$$

2) Write a sentence to explain what your answer means.

Part C How a Change in the Number of Flights per Year Affects the Minimum Stock Level (MSL) and the Order Quantity (OQ)

1) Suppose that the number of flights per year changes while other quantities such as lead time remain the same as given originally. Using the formula given previously for the Minimum Stock Level (MSL) fill in the following table.

Table 1

Number of Tires per Flight	Number of Flights per Year	Lead Time	Safety Pad	Minimum Stock Level (MSL)
	6			
	8			
	12			
	15			

2) Write a function to determine the Minimum Stock Level for **X** flights per year.

MSL = ______________________________________

3) Now write a function to determine the Order Quantity (OQ) for **X** flights per year, assuming other values (e.g. Lead Time, Safety Pad, Support Period, Due-In Quantity, Serviceable Quantity) remain the same. When you write your function, remember that the Order Quantity (OQ) depends on the Minimum Stock Level (MSL) which you have already expressed as a function of **X**. Remember to write the Order Quantity as a piecewise function since OQ = 0 when the MSL is less than the number of serviceable parts on hand (Ser.).

OQ = ______________________________if ________
OQ = ______________________________if ________

4. a) Graph the OQ function.
   b) What are the slopes of the two parts of the function? What do they mean in this situation?

5) Complete the following table. Assume that entries in columns 1, 3, 5, 6, and 8 remain the same.

Table 2

(1) Number of Tires per Flight	(2) Number of Flights per Year	(3) Safety Pad	(4) MSL	(5) Due-In Quantity	(6) Serviceable Quantity (Ser.)	(7) Order Quantity (OQ)	(8) Cost per Tire	(9) Total Cost
	6							
	8							
	12							
	15							

Part D How a Change in the Number of Tires on Hand Affects the Order Quantity (OQ)

Suppose that the number of tires on hand (Serviceable Quantity or “Ser.” in the Order Quantity formula) changes while the number of tires per flight (4 tires), number of flights per year (8 flights), Safety Pad (12 months), Lead Time (9 months), Support Period (12 months), and Due-In Quantity (0 tires) remain the same as given originally.

1) Write a *piecewise* function to determine the Order Quantity (OQ) when **X** tires are on hand. (Ser. = **X** in the formula.) Remember that if the Serviceable Quantity (Ser.) exceeds the MSL, you would not order any tires. (OQ = 0). This leads to a piecewise function.

   OQ = ______________________________________if ______________
   OQ = ______________________________________if ______________

2. a) Sketch the graph of your function.

   b) What are the slopes of the two parts of the function? What do they mean in this situation?

   c) What is the y-intercept? What does it mean in this situation?

3) Fill in the following table:

Table 3

Number of Tires On Hand (Ser.)	Order Quantity (OQ)
0	
27	
54	
81	

4) Functions can be represented by tables of input and output values, algebraic formulas, or graphs. In the context of questions about the relationship between order quantity and the number of tires on hand, what are some of the advantages and disadvantages of each of these representations?

Part E How a Change in the Safety Pad Affects the Minimum Stock Level (MSL) and the Order Quantity (OQ)

If NASA scientists and engineers are willing to accept more risk in having enough parts on hand they can use a Safety Pad less than the 12 month time period we have considered. **In this Part, we shall let the Serviceable Quantity (Ser.) be 20 rather than 54.** Assume that other quantities such as the number of flights per year and lead time do not change.

1) Write a function to find the Minimum Stock Level (MSL) for a safety pad of **X** months.

   MSL = ____

2) Now write a function to find the Order Quantity (OQ) for a safety pad of **X** months. (Remember that the Order Quantity depends on the Minimum Stock Level, and that the Order Quantity is zero when the Serviceable Quantity (Ser.) is greater than the MSL.)

   OQ = ____

3) Graph the functions for MSL and OQ as the safety pad, **X**, varies from 0 months to 12 months.

4. a) Describe the two graphs. What is the same? What is different?

   b) Use your functions and graphs to complete the following table:

Table 4

Safety Pad (months)	Minimum Stock Level (MSL)	Order Quantity (OQ)
0		
3		
6		
9		
12		

5) Use your Order Quantity function to determine the number of tires you would recommend ordering for a safety pad of 10 months.

6) In contrast with the Order Quantity functions for Parts B, C, and D, the OQ in this Part is not a piecewise function. Why is this?